Coupled-resonator micromechanical filters with voltage tuneable bandpass characteristic in thick-film polysilicon technology

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I ABSTRACT.

This paper reports on the design and characterization of 2.4 MHz and 9.2 MHz fourth and sixth order passband microelectromechanical coupled-resonator filters implemented in a 15 µm thick-film epitaxial polysilicon technology. The work uses a novel approach in the building of coupled-resonator electromechanical filter structures using on-line controlled electrostatic coupling. Mechanical springs commonly used to couple micro-mechanical resonators were replaced by electrostatic links created by pairs of biased electrostatical transducers. To control the coupling strength, an original biasing scheme of electrostatical transducers is proposed. The idea is to maintain a fixed charge on the floating-potential middle electrode of the electrostatical coupler. This electrode doesn’t need any external electrical connection, thus doesn’t suffer from parasitic capacitances.

The coupling factor is directly controlled by coupling transducer bias voltages, allowing the control of the filter pole frequencies. A voltage controlled bandpass filter at 9.2 MHz showed a bandwidth with a tuning range of 3.18 to 47.4 kHz.

Keywords: microelectromechanical filter, coupling, tunable, electrostatical transducer, thick-film

II INTRODUCTION AND STATE OF THE ART

A keen interest has been shown recently in micromachined mechanical silicon integrated filters for analog signal processing in sensing and telecommunication applications [1], [2]. The challenges represented by the realization of silicon micromechanical passband filters have been highlighted in numerous studies touching upon technology, design and performance issues. Considerable efforts have been devoted to either design or improvement of second-order electromechanical resonators which constitute the building blocks of electromechanical filters [3-6].

High-order electromechanical filter design involves several elementary resonators interacting with each other in a way to generate a given passband function. Customarily, coupling of mechanical resonators is achieved with a soft mechanical beam connected between two resonators. This technique was used to implement filters with three resonators at 450 kHz [7] and with two resonators at 70 MHz [8]. Also, an alternative to mechanical coupling beam has been proposed [9]: twenty 5 MHz drumhead resonators has been coupled thanks to partial overlapping of the drumhead vibrating areas.

However, mechanical coupling approach presents several limitations concerning the geometry of the filter (in the most cases only adjacent resonators can be coupled), the transfer function (only “all-pole” functions are achievable) and the tuning (the coupling strength can’t be modified after fabrication).

Recent works put forth an alternative technique for coupling of mechanical resonators. An elastic link between closely spaced resonators is introduced by electric field which is generated by voltage applied between the resonators [10]. This approach offers several advantages, namely, voltage control of the elastic link (hence a good filter tunability) and the absence of a coupling spring. This technique has been used to realize a fourth-order filter composed from two clamped-clamped beam resonators coupled using this technique.

However, the need to closely space the vibrating elements of the resonators is very restricting. From the geometrical shape of the resonators [10] it appears that the electrostatic coupling can seldom be used with three or more clamped-clamped beam resonators. Moreover, the technique can’t be directly applied to resonators with non-rectilinear shape (e. g. vibrating disk resonator).

Structures in which an intermediate DC-isolated coupling electrode inserted in-between the vibrating elements have been presented recently [11]. The coupling electrode is placed close to the vibrating elements, in a way to yield two series-connected electrostatical transducers. It can have an arbitrary shape, thus liberating the designer from the geometrical restrictions mentioned above. The coupling strength is fixed at the design level by the value of the common node parallel-to-ground capacitance. The passband shape is tuned by adjusting the resonator bias voltage, which doesn’t affect the coupling factor but only the natural frequency of resonators. This tuning technique increases the insertion loss and can only be used for small passband shape correction.

A high-range tuning of the bandwidth is only possible if the coupling strength is controlled. Our study shows that the latter is defined by DC biasing conditions of the coupling electrode. However, the biasing of this electrode and its influence on the structure behavior haven’t been investigated yet. The DC potential of this electrode depends on voltages applied to resonators and on the electrode electrostatic charge. This charge hasn’t been taken into consideration up to now. If its value isn’t controlled, a mismatch may exist between the real and the designed filter performance.

This paper presents a complete theory of electrical coupling which takes into account the electrical charge of the coupling electrode. This theory is applied to design of variable-bandwidth electrically-coupled micromechanical filters. An original biasing scheme is used for control the charge of the coupling node and hence the filter bandwidth. Fabrication and test results of filters with two and three electrically coupled resonators are discussed in the paper.

The article is organized in the following way. In section III (theoretical part) a brief review of the basic theory of coupled resonators is given. Then it is shown how to obtain the described coupling device from the basic close-spacing electrostatical coupler presented in [10]. Its operation, theory and equivalent mechanical model are described. Section IV is devoted to the realization and
testing of filters with the proposed approach. Two-resonator filters were realized at frequencies of 2.4 MHz and 9.2 MHz. A filter with three resonators was only implemented at 2.4 MHz. Experimental and simulation results are presented and compared.

III THEORY OF ELECTROSTATIC COUPLING

A. Theory of coupled resonators

Coupled-resonator filters are built from several identical resonators interacting through a reactive link [12]. Without coupling, such a system has two pairs of finite conjugated poles situated at the resonator natural resonance frequency 1. Non-zero coupling between the resonators results in poles splitting around the resonator natural frequency. For example, this happens when RLC resonators are magnetically coupled.

In mechanical filters the coupling link is elastic and generally realized with mechanical springs, as shown in fig. 1a for the case of a two resonator lumped-element system [13]. Input and output transducers needed to interface with electrical-domain signals are not shown. In the mechanical domain the input value is the force acting on one of the resonators (or on a mass in the lumped-parameter representation), the output value is the velocity or displacement of the other resonator (its lumped mass).

Such a system has two conjugated pole pairs. Fig. 2 shows the evolution of the corresponding frequencies \( f_{p1,2} \) and \( f_{p3,4} \) when the coupling strength increases. Pole separation is observed: the frequency of one of the pole pairs increases whereas the other pole pair frequency is constant and remains equal to the natural frequency of the resonators. Thus the center frequency increases with the coupling. However, in the classical theory of coupled resonator the coupling link doesn’t affect the center frequency: the latter remains equal to the resonance frequency of the isolated resonators. Nevertheless, a classical theory can be used here if we introduce the notion of individual resonator in the context of filter which we will call filter individual resonators. They are composed from the original (physical) resonators and from the coupling element. One of them can be put to the fore if one mass of the diagram fig. 1a is fixed (fig. 1b).

From the fig. 1b, the natural frequency of the filter individual

\[
\begin{align*}
\text{Original resonators} & \quad \text{Coupling spring} & \quad \text{Filter individual resonator}
\end{align*}
\]

\(\text{a) } \quad \text{b)}

Fig. 1. System including two coupled resonators: a) global structure, b) structure of a filter individual resonator.

1. We call the frequency of a pole the absolute value of the pole. If the system is weakly damped (which is the case of all our resonators), it corresponds to the frequency at which appears the maximum of the transmission characteristic corresponding to this pole, i. e. the resonance frequency.
This equation can be limited and expressed as:

Modeled by lumped viscous dampers $\mu_0$, original resonator damping is another important parameter after the poles position which defines the passband shape, in particularly, the passband ripple amplitude. It can be shown that the maximally-flat passband can be obtained when the coupling factor $\alpha$ is equal to the inverse of the quality factor of the filter individual resonators. This value of $\alpha$ is called “critical coupling factor”: at higher values a passband ripple appears, lower values yield an increase of transmission loss weakly affecting the passband shape [12], [14]. It can easily be shown that in the case of critical coupling, the bandwidth of the corresponding Butterworth filter equals $\sqrt{2}$ times the offset $\Delta f$ between the filter passband pole frequencies [15].

In a silicon micromachining technology the elastic coupling can be implemented with mechanical spring or with electrostatic transducers. However, the mechanical approach has several drawbacks: a complex behavior of the coupling element, difficulties of design and simulation, the impossibility of tuning. In addition, mechanical coupling is often difficult to use with high-frequency resonators (>100 MHz). To show it, let consider the following example. A filter with overall quality factor of 1000 has a ratio of 1000 between the stiffness of individual resonators and the coupling spring. Knowing that $k \sim (W/L)^3$, where $W$ and $L$ are the width and length of a beam [13], the ratio between linear dimensions of the resonator and of the coupling beam should approach 10. A 1.0 $\mu$m large high-frequency resonator requires a 0.1 $\mu$m large coupling beam, which can be hard to achieve with conventional micromachining technologies.

B. Electrostatic coupling principle

Electrostatic coupling is directly obtained by replacing a mechanical coupling spring $k_C$ with a simple DC biased parallel-plate capacitive transducer as shown in fig. 3 where input-output transducers are omitted for clarity. This coupling technique was used to realize a two-resonator filter in [10] (fig. 4). Being the starting point for more complex coupling structures, we resume here the theory of this device.

Since the transducer is biased by a DC voltage, a DC electric field exists between the transducer electrodes, generating DC forces on them. Motion of one of the resonators modifies the field exists between the transducer electrodes, generating DC forces on them. Motion of one of the resonators modifies the transducer capacitance hence generates a variation of the forces acting on both resonators. A mechanical coupling appears in this way. In fig. 3 the bias voltages are ground-referenced in order to avoid dealing with floating voltages.

Instantaneous force and displacement values $f_1$, $f_2$ and $x_1$, $x_2$ are related as

$$f_1 = f_2 = \frac{(E_1 - E_2)^2}{2} \frac{S}{\varepsilon_0 (d_0 - (x_1 - x_2))^2}, \quad (7)$$

where $d_0$ is the initial distance between the transducer electrodes (gap) and $S$ is the transducer area. For small displacements this equation can be limited and expressed as:

$$f_1 = f_2 = \frac{(E_1 - E_2)^2}{2} \frac{S}{\varepsilon_0 d_0}, \quad \Delta f = \frac{(E_1 - E_2)^2}{2} \frac{S}{\varepsilon_0 d_0}.$$

Fig. 3. Principle of electrostatic coupling.

Fig. 4. Possible configuration of two clamped-clamped beam resonators coupled by a parallel-plate capacitive transducer.

$$\begin{align*}
F_1 &= \frac{\partial f_1}{\partial x_1} \frac{\partial f_1}{\partial x_1} \times x_1 = \left[-k_{eq} - k_{eq} \times X_1 + X_2 \right], \\
F_2 &= \frac{\partial f_2}{\partial x_2} \frac{\partial f_2}{\partial x_2} \times x_2 = \left[-k_{eq} - k_{eq} \times X_1 + X_2 \right], \quad (8)
\end{align*}$$

where $F_1$, $F_2$, $X_1$ and $X_2$ are the incremental values of the forces and the displacements, $k_{eq}$ equals:

$$k_{eq} = -(E_1 - E_2)^2 \frac{S}{\varepsilon_0 d_0}. \quad (9)$$

It is easy to see that equation (8) describes a linear mechanical spring with stiffness $k_{eq}$ having end coordinates $x_1$ and $x_2$. Thus, a capacitive transducer in small-signal displacement mode is equivalent to a mechanical spring and can be used to couple mechanical resonators. Note that the resonator voltages control both the coupling strength and the resonator natural frequency. Thus an independent tuning may require additional (non-signal) transducers [10].

As can be seen from fig. 4, this coupling technique is not suitable for implementation of filters with higher number of clamped-clamped beam resonators, since the coupling transducer is formed by the walls of the resonating elements which should be placed close to each other.

C. Introducing of electrostatic coupling with coupling node

To obtain a more practical structure, let us suppose that an electrically neutral and isolated conductor plane is inserted into the parallel-plate capacitor (fig. 5a): it doesn’t modify the electrostatic configuration. If the plane is split into two electrically connected planes, we obtain again an equivalent electrostatic system (fig. 5b), since the electrostatic field is still localized inside the
obtained transducers. Thus, the system of fig. 5b behaves in the same way as the original coupling capacitor (fig. 3), but allows a free resonator placement.

The operating on this device is based on the exchange of electrical charge between the transducers of coupler. Therefore, the coupler is very sensitive to the parasitic capacitances associated to the common node (later we will call it “coupling node”). To take it into consideration, the diagram fig. 5b incorporates the parasitic parallel-to-ground capacitance $C_p$ of the coupling node.

A realization of two-resonator filter with such an architecture has been shown recently [11]. In the next section we present a complete analysis of this structure and show how a coupling strength control can be achieved in such a system.

D. Charge biasing and device analysis

In section III.C it was assumed that the coupling node was electrically neutral. To be valid in practice, this assumption requires a use of a biasing technique allowing a control of the common node electrical charge (the latter can also be, generally speaking, set to non-zero values). In this section we propose an original technique allowing such a control, and analyze the operating of the obtained device.

A straight way of the common node biasing is the use of a high-value choke inductor or resistance which connects the common node to a DC voltage source: nor AC neither DC current would circulate in this element, the charge of the node would remain constant. However, high-value inductors or resistors are difficult to implement in integrated technologies. Off-chip elements are undesirable because of the tremendous increase of the parasitic capacitance $C_p$ on the coupling node (a high $C_p$ results in a dramatic loss of the coupling strength [11]).

We developed a new charge-biasing scheme in which the coupling node is connected to a DC voltage source $E_{12}$ by a micro-mechanical switch which is closed only for the time needed to charge the coupling node transducer and parasitic capacitance $C_p$ (fig. 6). These capacitances are small (~100 fF), so even if the switch contact resistance (in «on» state) is very high (megohms), the time constant is very low and the node sets quickly to $E_{12}$. During this setup phase no signal is applied, the resonators don’t vibrate, the transducers capacitances are at their quiescent values. After the setup phase the switch is opened (off), the system can operate in filtering mode with a non-zero input signal. Since the node is isolated, the charge of the node remains constant.

The following analysis of coupler operation in filtering mode assumes that the switch is open («off»), and that during the initialization phase (i.e. closed switch), the static DC voltage of the coupling node has been set to $E_{12}$. We expose here the principal steps of the device analysis assuming that $E_{12}=0$. This doesn’t affect the generality of demonstration since such a configuration can always be obtained by an appropriate choice of the reference voltage level. The analysis starts from calculation of the coupling node charge ($Q_0$). It is defined when the switch is closed (setup phase):

$$v_1=E_1-E_{12}=E_1, \quad v_2=E_2-E_{12}=E_2, \quad v_{12}=E_{12}=0,$$

where $v_1$ and $v_2$ are the instantaneous values of transducer voltages, $v_{12}$ is the instantaneous value of the coupling node voltage. Thus, the charge of the coupling node $Q_0$ equals the sum of the charges of these three capacitances:

$$Q_0 = E_1C_{10} + E_2C_{20}, \quad (10)$$

where $C_{10}$ and $C_{20}$ are the quiescent (at $x_{1,2}=0$) capacitances of the transducers.

The operation of this device obeys to the Kirchoff law for the mesh $E_1-E_2-C_2-C_1$ and to the charge conservation law for the
common node:

\[
\begin{align*}
    v_1 &= E_1 - v_{12} \\
    v_2 &= E_2 - v_{12} \\
    Q_0 &= v_1 c_1 + v_2 c_2 - v_{12} C_p
\end{align*}
\]  

(11)

where \(c_1\) and \(c_2\) are the instantaneous values of the transducer capacitances.

From the equations (10) and (11) we can express the instantaneous value of voltage \(v_{12}\):

\[
v_{12} = \frac{E_1 \cdot (c_1 - C_{10}) + E_2 \cdot (c_2 - C_{20})}{c_1 + c_2 + C_p}.
\]

(12)

The voltages \(v_1\) and \(v_2\) are directly calculated from (11)a, b and (12).

From (11)a, b and (12), the relation between the instantaneous values of forces acting on the resonators \(f_1\) and \(f_2\) and the displacements \(x_1\) and \(x_2\) can be found [15]:

\[
\begin{align*}
    f_1 &= \frac{1}{2} \frac{d^2 dc_1}{dx_1^2} \cdot c_i = e_0 \frac{S_i}{d_{10} - S_i}, i = 1, 2, \\
    f_2 &= \frac{1}{2} \frac{d^2 dc_2}{dx_2^2}
\end{align*}
\]

(13)

d_{10} and \(d_{20}\) are the transducer gaps, \(S_1\) and \(S_2\) are the transducer areas.

A limited development of (13) (similarly with (8)), using (11)a, b and (12), gives the following small-signal matrix equation:

\[
\begin{bmatrix}
    F_1 \\
    F_2
\end{bmatrix} =
\begin{bmatrix}
    \frac{E_1^2 C_{10} (C_{10} + C_p)}{d_{10}^2 C_{10} + C_{20} + C_p} & \frac{E_1 E_2 C_{10} C_{20}}{d_{10}^2 C_{10} + C_{20} + C_p} \\
    \frac{E_1 E_2 C_{10} (C_{10} + C_p)}{d_{10}^2 C_{10} + C_{20} + C_p} & \frac{E_2^2 C_{10} (C_{10} + C_p)}{d_{10}^2 C_{10} + C_{20} + C_p}
\end{bmatrix}
\begin{bmatrix}
    X_1 \\
    X_2
\end{bmatrix}.
\]

(14)

where \(F_1\), \(F_2\), \(X_1\) and \(X_2\) are the small-signal magnitudes of the transducer forces and resonator displacements.

Interpretation of the obtained result and corresponding mechanical model are presented in the next section.

E. Mechanical and electrical model of the coupler

The matrix equation

\[
\begin{bmatrix}
    f_1 \\
    f_2
\end{bmatrix} =
\begin{bmatrix}
    y_{11} & y_{12} \\
    y_{21} & y_{22}
\end{bmatrix}
\begin{bmatrix}
    x_1 \\
    x_2
\end{bmatrix}
\]

(15)

describes a lumped-parameter mechanical system including two mobile nodes (with number 1 and 2). The nodes are linked with each other and with the fixed reference by elastic links (fig. 7).

The convention about the spring subscripts is the following: all equivalent springs associated with the system are designed by the numbers of the mechanical nodes involved in the system. In addition, the subscript of a side spring (connected between a node and the fixed reference) includes twice the number of the node to which the spring is connected (cf. fig. 7).

A simple mathematical transformations allow to find the corre-
IV DESIGN AND REALIZATION OF COUPLED-RESONATOR FILTERS IN THICK-FILM TECHNOLOGY

A. Presentation of the individual resonators.

Fig. 8 shows a simplified isometric view of one of the resonators used in the designed filters. The resonators are implemented in 15 \(\mu\)m thick epitaxial polysilicon technology. This process uses a 15 \(\mu\)m thick polysilicon structural layer for active part realization and a 0.45 \(\mu\)m thick buried polysilicon layer for anchoring and interconnection of the structural layer parts. Vibrating beam and input-output electrodes are realized in thick structural layer; the beam is designed to vibrate in the fundamental lateral flexural mode. Detailed information about operation principles and realization of such resonators can be found in [16].

The technology doesn’t allow the realization of a submicron transducer gap, necessary to obtain resonators with a reasonable impedance level (below 100 k\(\Omega\)). To circumvent this restriction, a post-fabrication autoassembling gap reduction technique was developed for these resonators [16]. As can be seen from the photo of single resonator (fig. 9), the input-output electrodes are attached to a rigid bar which is joined to a soft spring. All these elements are released and can move in the lateral plane. A pair of additional electrostatic transducers (motor transducers) allows, when biased, to displace the rigid bar so to reduce the signal electrode-to-beam distance (signal transducer gap). This displacement is limited by the stoppers whose position controls the actual gap value.

When a resonator is used (as part of a filter or as independent device), the gap reducing mechanism should be activated. For this the electric potential of the motor electrodes should be maintained at least 25-30 V above the potential of the signal electrodes (or below, since the force is sensitive to the square of voltage). Note that the signal electrodes are electrically connected with the rigid bar, since machined in the same layer.

Figure 10 presents the measured characteristic (resonance frequency against bias voltage) of resonators having the same geometry as the devices used in the realized filters: a 81.2 \(\mu\)m long 1.8 \(\mu\)m wide and a 41.2 \(\mu\)m long 1.8 \(\mu\)m wide clamped-clamped beam resonators. From the plots, the intrinsic resonance frequencies (at \(E_0=0\)) are 2.32 MHz and 10.85 MHz [8].

However, the resonators used in filters exhibited slightly different resonance frequencies (2.4 and 9.2 MHz): having identical designed geometry, they were fabricated in different technological runs with some process parameters modified [16].

Another very important resonator parameter is the motional resistance (\(R_X\)). It is equal to the resonator impedance at the resonance frequency. At \(E_0=20\) V the 10.8 MHz resonator had a
motional resistance of 142 kΩ, whereas the 2.3 MHz resonator had a motional resistance of 75 kΩ. The motional resistance is inversely proportional to the square of the bias voltage.

B. Filter implementation

The coupling node biasing technique presented in section III.D was used to implement two and three resonator filters. A three-resonator filter whose simplified diagram is given in fig. 11a was designed with 2.4 MHz resonators. Similar structures included two resonators were designed with both 2.4 MHz and 9.2 MHz resonators. Photos of one of the realized filters are showed in fig. 11. Dimensions and main filter parameters are presented in table 1.

The electrostatic transducers employed in the realized filters used the gap reduction mechanism, in order to achieve a submicron gap in all transducers. The gap reduction mechanism should be actuated all time during the filter operation.

The micromechanical switch has the same geometry as the gap reduction motors, with only difference that the contact electrode gets into mechanical and electrical contact with the common node when the mechanism is actuated.

C. Filter modeling

Filter was modeled with Spectre electrical circuit simulator [17] in Cadence Design Systems environment. Vibrating elements in fundamental mode were modeled by their equivalent electrical networks [14], [18-20]; electrostatic transducers were modeled with Spectre Analog Hardware Description Language (AHDL) using classical electrostatic transducer theory [15], [18]. The parameter values used in the simulation are given in the table 1.
Table 1: Summary of the parameters of the tested resonators.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>2.33 MHz filter</th>
<th>9.2 MHz filter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length, µm</td>
<td>81.2</td>
<td>41.2</td>
</tr>
<tr>
<td>Width, µm</td>
<td>1.8</td>
<td>1.8</td>
</tr>
<tr>
<td>Height, µm</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>Vibrating element resonance frequency, MHz</td>
<td>2.6</td>
<td>9.6</td>
</tr>
<tr>
<td>Resonator stiffness (simulated with Coventor Ware), Nm⁻¹</td>
<td>490.5</td>
<td>3671</td>
</tr>
<tr>
<td>Maximal quality factor (low bias voltage)</td>
<td>17000</td>
<td>2500</td>
</tr>
<tr>
<td>Designed gap width (when gap reduction actuated), µm</td>
<td>0.4</td>
<td>0.2</td>
</tr>
<tr>
<td>Effective (extracted) gap width (gap reduction actuated), µm</td>
<td>0.629</td>
<td>0.31</td>
</tr>
<tr>
<td>Transducer’s length</td>
<td>38.8</td>
<td>18.8</td>
</tr>
<tr>
<td>Measured resonator motional resistance at $E_{0}=20$ V, kΩ</td>
<td>37.3</td>
<td>142.3</td>
</tr>
<tr>
<td>Absolute resonator resonance frequency tuning range $f_{\text{max}}/f_{\text{min}}$ for $E_{0}=0...50$ V, kHz</td>
<td>356.3</td>
<td>472.6</td>
</tr>
</tbody>
</table>

D. Filter dimensioning and tuning issues

The realized filters have the ladder coupled-resonator architecture, therefore they possess “all-poles” transmission characteristic [12]. The center frequency is roughly determined by the individual resonator resonance frequency, i. e. by the resonator geometry (the biasing allows only to adjust the resonator resonance frequency in limits of few percents, as shows the plot fig. 10). However, the pole distribution in the passband is mainly defined by the coupling factor between the resonators; the passband smoothing depends on the resonator damping (cf. section III.A). Both parameters can be set after the filter fabrication, the first with the appropriate values of the bias voltages $E_{1}$, $E_{3}$, $E_{12}$ and $E_{23}$, the second by an appropriate choice of the termination resistors $R_{\text{load}}$ and $R_{\text{source}}$. To obtain the transmission characteristic with given specifications (bandwidth shape, ripple etc.), a numerical approximation method should be employed in general case. For widely used and well-studied filter architectures, standard design procedures exist which, based on normalized parameter tables, can be applied to filters with any physics [12]. We present now how to calculate the control parameters of our structures using such a dimensioning method given in [12] for ladder coupled-resonator filters.

Dimensioning of a ladder filter composed from $L$ electrostatically coupled resonators (each resonator being coupled with no more than 2 resonators), with electrostatic transducers at input and output (fig. 11a) implies three conditions:

(i) Matching of resonance frequencies of the individual resonators $f_{i}$ with the desired filter center frequency $f_{C}$. It gives $L$ equations depending on all bias voltages. For the edge resonators with subscripts 1 and $L$, the frequencies $f_{1}$ and $f_{L}$ are given by:

\[
f_{i} = f_{C} \cdot \frac{1}{1} \left[ \frac{k_{12} \text{ if } i=1}{k_{0} \text{ if } i=L} + \frac{k_{112} \text{ if } i=1}{k_{0} \text{ if } i=L} \right] = f_{C}, \tag{19}
\]

where $k_i$ is the stiffness of the electrostatic spring generated by the input (output) transducer depending on the resonator bias voltages $E_i$ ([14], [16]), $k_{12}$, $k_{112}$, $k_{(i-1)i}$ and $k_{(i-1)iL}$ are the stiffnesses of the electrostatic springs generated by the edge couplers depending on $E_i$, $E_j$ and on the bias voltages of the edge coupler transducers $E_{12}$ and $E_{(L-1)L}$ (cf. eq. (17) or (18)). This formula expresses the natural frequency of a resonator with stiffness $k_i$ to which are connected three springs [14]: one formed by the input or output transducer ($k_i$) and two formed by the coupler which links this resonator with its neighbor (i. e. one side spring and one coupling spring).

For $i=2...L-1$ (the non-edge resonators) the resonance frequencies are determined by the stiffneses of the side springs $k_{(i-1)i}$, $k_{(i-1)iL}$, $k_{i(i+1)}$ and $k_{i(i+1)L}$ generated by the couplers adjacent to the resonators:

\[
f_{i} = f_{C} \cdot \frac{1}{1} \left[ \frac{k_{(i-1)i} + k_{(i-1)iL}}{k_{0}} - \frac{k_{i(i+1)} + k_{i(i+1)L}}{k_{0}} \right] = f_{C}, \tag{20}
\]

This condition generates $L$ independent equations.

(ii) Normalized coupling factors of all couplers should equal the values $k_{ij}/k_{ij,norm}$ given in the filter cookbook [12]:
2.435
2.372
2.445
78
2.368
2.37
2.4
2.374

adjusting the resonator bias voltages (up to 5 \%) [15]. This mismatch was corrected by the resonance frequencies of individual resonators due to fabrication tolerances. As it was predicted by the theory (section III.D), the voltages for the termination resistors.

where $Q_i$ is the effective quality factor of the resonator $i$, $q_i$ is the normalized resonator quality factor given in the filter design table, $q_0$ is the quality factor of the resonating element, $R_i$ is the external termination resistor, $L_X$ and $R_X$ are the motional parameters of the resonators. Note that the latter depends on the bias voltages $E_1$ and $E_L$ defined from the rules (i) and (ii) [14].

The three rules generate $2L-1$ equations for $2L-1$ bias voltages and 2 equations for 2 termination resistors. Numerical resolution of these equations allows to dimension a filter with desired transfer function. An example of dimensioning of a three-resonator filter will be given in section IV.F.

E. Test results of two-resonator filter

The designed devices were tested under a 10⁻¹ Torr air pressure, at which the air damping was negligible. The filter was terminated by a 1 kΩ resistance at the output and by 25 Ohms resistance at the input. The output voltage was amplified by 10 with a voltage amplifier. The same test setup was used for the test of the three-resonator filters. The goal of the experiments was to demonstrate the voltage control of passband pole frequencies.

For all tested devices we observed a mismatch of intrinsic resonance frequencies of individual resonators due to fabrication tolerances (up to 5 \%) [15]. This mismatch was corrected by adjusting the resonator bias voltages $E_1$ and $E_2$.

The goal of the first experiment was to demonstrate the evolution of the filter transmission characteristic with voltage $E_{12}$. A family of frequency transmission characteristics was obtained for 2.4 MHz filter using different voltages $E_{12}$ (fig. 12). Before each measurement the coupling node was pre-charged with new $E_{12}$ by closing the switch; during the measurement the switch was opened. As it was predicted by the theory (section III.D), the voltage $E_{12}$ controls the offset between the passband pole frequencies. During this experiment the resonator damping remained constant since neither termination resistors nor air pressure changed. According to the theory [12], the passband ripple increases for high coupling factors. A modeling of this experiment results in similar curves (fig. 13). The maximal value of $E_{12}$ (thus minimal coupler bias voltages $E_1-E_{12}$ and $E_2+E_{12}$) corresponds to the configuration with the critical coupling factor, i.e.

\[
E_1=2.5 \text{ V, } E_L=23.85 \text{ V} \\
Af = 1940 \text{ Hz}
\]

\[
E_{12}=14.0 \text{ V, } E_2=27.2 \text{ V} \\
Af = 426 \text{ Hz}
\]

Fig. 12. Family of measured characteristics for different $E_{12}$ obtained for 2.4 MHz two-resonator filter.

to a Butterworth passband.

Both the simulation and the experiment show that the filter center frequency increases with $E_{12}$. This frequency drift is generated by the springs $k_{112}$ and $k_{122}$ of electrostatic coupler whose negative stiffnesses decrease their absolute values when $E_{12}$ increases (cf. eq. (18), (19), (20)).

A similar experiment was carried out with a 9.2 MHz filter, with the only difference that for each $E_{12}$ the resonator quality factor was adjusted by the air pressure in order to yield a Butterworth passband shape (fig. 15). The observed loss of transmission for large bandwidths can be explained by the fact that the resonator damping was realized by an increase of the motional resistance $R_X$ rather then by an increase of the termination resistor values. The high motional resistances of the resonators and the relatively small load resistor form a voltage divider increasing the loss in the passband.

Correct impedance matching and filter quality factor control would required load resistors with values much higher than the motional resistances (i.e. hundreds kilo-ohms). Such resistors would be shunted by the parasitic capacitances of the measurement setup (PCB and pad capacitors, total value ~1-2 pF) [10].
The second experiment aimed to characterize the tuning potential of the filter. \( E_{12} \) was swept from zero to its maximal value (corresponding to the critical value of the coupling factor). Each characteristic was described by the frequency offset between the maxima of the transmission characteristic. In this way a plot “maxima frequency offset against voltage” was obtained (fig. 14). It can be considered that the frequencies of the maxima coincide with the frequencies of the poles of transmission characteristic. An exception, however, is when the relative maxima frequencies of the poles of transmission characteristic (fig. 14). It can be considered that the frequencies of the maxima “maxima frequency offset against voltage” was described by the frequency offset between the corresponding bias voltage products (\( E_{12} \)). Each max-to-min ratio of the pole frequency offset is equal to the max-to-min ratio (given in table 1), and we fixed the following target specifica-

![Fig. 14. Passband pole frequency spacing against \( E_{12} \): measured and simulated evolutions for 2.4 MHz filter.](image)

\( \Delta f_{max} / \Delta f_{min} = \frac{506}{575} = 8.8; \)

whereas \( \Delta f_{max} / \Delta f_{min} = 2479 \text{ Hz} / 278 \text{ Hz} = 8.9 \), which is in a very good agreement with the theoretical value. For 9.2 MHz filter the bias voltage product ratio is 8.43 whereas \( \Delta f_{max} / \Delta f_{min} = 14.9 \), i.e. 1.76 times higher. This discrepancy can be explained by the relatively low quality factor of the 9.2 MHz resonators, which, when the coupling is weak, yields the pole frequency offset higher than the maxima frequency offset. However, the frequency offset values observed on fig. 16 with high coupling, for example, at \( E_{12} \) of 15 V and 0 V, are perfectly conformal to the theory.

The most relevant information about the filter coupling capability is given by the pole frequency offset at the maximal coupling factor value (which is limited by the maximal bias voltage supported by the transducers) since it fixes the maximal filter bandwidth. The low limit of the bandwidth is fixed by the resonator quality factor (which is limited by the resonator geometry and technological parameters) rather than by the coupling factor (which can easily be reduced).

**F. Three-resonator filter test results**

Theoretical study based on the coupled-resonator filter design tables [12] showed that, in contrast with two-resonator filters, (i) dimensioning of a regular passband shape three-resonator filter is not symmetrical regarding to the middle resonator (\( E_1 \neq E_3, E_{12} \neq E_{23}, R_{p} \neq R_{r} \)) (ii) regular passband shape can only be obtained with specific filter terminations (with contrast from two-resonator filters, where equal transmission levels at maxima were observed even with unsymmetrical terminations) [9]. To find these specific parameter values, we used the method presented in section IV.D. We took the known parameters of resonators and transducers (given in table 1), and we fixed the following target specifications: Butterworth characteristic, 2.28 MHz center frequency, overall quality factor of the filter four times lower than the resonator quality factor (10 000). To fix the control parameter values, a system of 7 unknowns possessing 6 physically different root vectors was solved. Table 2 present one of the obtained root set. These values applied to the Spectre model of the filter gave precisely the aimed transmission characteristic. All the valid root sets yielded the same simulated characteristic conformed to the specifications.

However, in experiment, these biasing parameters didn’t yield a regular Butterworth characteristic, mainly because of random dispersion of the resonator parameters (about 5%) due to fabrica-
tion tolerances. Fig. 17 presents three transmission characteristics of three-resonator filters we observed after manual adjusting of the bias voltages. They have an irregular passband shape. In fact, given a big number of the tuning parameters and a high sensitivity of the characteristic toward bias voltages and toward termination resistances, also because of parasitic capacitances at input and output, it was impossible to obtain a correct passband shape by «trial and error». Issues related to tuning are discussed in the next section.

![Transmission Characteristic](image)

**Fig. 17.** Family of measured characteristics for three-resonator 2.4 MHz filter.

**G. Tuning issues of high-order filters**

Two parameters can characterize the tuning potential of a high-order coupled-resonator filter.

The first is the sensitivity of the resonance frequency of the individual resonators with respect to the resonator bias voltages $E_i$. As we could see in section III.D from (18), a coupler with high $C_p$ creates the same resonance frequency shift as a single transducer biased by voltage $E_i-E_{i+1}$. Thus, to estimate the sensitivity, one can consider the «voltage - resonance frequency» characteristic of an isolated resonator with the same geometry biased by $E_0$ (fig. 8). The sensitivity increases with decrease of the resonator mechanical stiffness and with increase of the coupling between the resonator and the input-output electrodes (transducing factor). This sensitivity can be defined as:

$$a_{\text{run}} = \frac{df_0}{dE_0}$$  \hspace{1cm} (24)$$

where $f_0$ is the resonance frequency of a biased resonator ([14], [16]). As it can be calculated from fig. 10, for 10 MHz resonator, at 30 volts bias voltage $a_{\text{run}}$ equals 11.4 kHz/V. This value can be compared to the bandwidth of 10.8 kHz. If a precision of 1% bandwidth is required for definition of the filter pole frequencies, voltages $E_i$ should be set with 10 mV resolution.

The second tuning parameter represents the sensitivity of the pole frequency offset with respect to the voltage $E_{i+2}$. Its value can be found from the plots fig. 14 and 16. From formulae (4), (6) and (20), assuming that the resonators have identical geometry and are biased by equal bias voltages $E_1=E_2=E_0$, this sensitivity can be expressed as:

$$a_{\text{diff}} = \frac{d\Delta f}{dE_{12}} = \frac{f_0^2 k_{12}}{k_0 dE_{12}} = -\frac{E_0 - E_{12}/R}{d^2} k_0^2 2C_0 + C_p$$  \hspace{1cm} (25)$$

If $C_p$ is high, this sensitivity is much lower than the sensitivity defined by (24) (this can be seen comparing the slopes of fig. 16 and fig. 10). Nevertheless, as $C_p$ decreases, $a_{\text{diff}}$ increases and tends to become comparable with $a_{\text{run}}$.

Practical use of such filters can only be envisioned with automatic tuning system. However, given the number of control parameters for high-order filters and insufficient knowledge about the errors on resonator performance, the automatic tuning can be a very complex and costly task. To simplify the tuning, the nominal (a priori) parameter values (which can be stored in a look-up table) should approach the really needed values, so that the tuning corrects only small remaining errors. To allow a good prediction of the initial parameter values, the fabrication tolerances should be minimized, which requires a substantial improve of the fabrication accuracy.

**V. CONCLUSIONS AND PERSPECTIVES**

This work demonstrates a new approach to realization of high-order tunable micromechanical filters employing electrostatically-driven micromechanical resonators. The electrostatic coupling technique frees designers from the problems associated with mechanical coupling beams such as distributed mass and losses and restrictions regarding the resonator placement. The original concept of coupling node biasing allows to control the frequency of poles and zeros defining the passband shape. This concept is realized using a novel technique of fixed-charge biasing of the coupling node. Fixed-charge biasing allows to control the electrical charge of the coupled node without external connection to the latter, thus minimizing parasitic capacitances and increasing the tuning range.

Two fourth-order filters were presented: one with 9.2 MHz center frequencies having 10-28 kHz bandwidth tuning range and one with 2.4 MHz center frequency having 300-2500 Hz bandwidth tuning range. These devices were implemented in a thick-film polysilicon technology which was firstly used for high-order high-frequency bandpass filters.

Theoretical analysis of the electrostatic coupling is exposed; it is shown that the demonstrated devices have a high tuning potential allowing to program the filter pole and zero frequencies. Physical model of the device has been developed.

The limits to the demonstrated filter performances, namely, relatively low center frequency and quite high impedance, have been determined by the properties of the used thick-film polysilicon technology and not by the technique of coupling. In fact, the presented concept can be used as well with high-frequency high-stiffness downscaled resonators. For such resonators, the maximal available coupling factor $\alpha$ (cf. eq. (2) or (4)) tends to decrease: small transducer dimensions lead to a drop of the electrostatic spring stiffness, whereas the resonator stiffness increases (this increase is due to the geometry and the vibration modes used by
high-frequency resonators). However, the coupling factor should at least equal the inverse of the intrinsic quality factor of the resonators (cf. section III.A). To compensate resonator stiffening, the coupling spring stiffness should be increased. Transducer gap reduction is the most efficient way to achieve this. We calculated, that to couple efficiently two 1.14 GHz radial-mode disk resonator presented in [6], having stiffness of 73 500 000 Nm$^{-1}$, the coupling transducers should have a ~10 nm gap. Although presenting a technological challenge, such a nanometric transducer gap can be compatible with very weak-magnitude vibration of high-frequency volume vibration mode resonators.

Electrostatic coupling is also possible between more than two resonators, allowing architectures with complex topology. For example, a T-filter having a characteristic with transmission zeros in the stopband is possible, as well as multi-stage lattice filters [12], [21]. Generally speaking, using the charge biasing of high-impedance signal nodes, all existing architectures based on quartz or BAW resonators can be reproduced with electrostatically-driven micromechanical resonators.

Integrated on silicon with active electronic circuits, exhibiting low impedance hence needing relatively low matching resistors, allowing bandwidth reconfiguration — with future SOC technology evolution such filters can become attractive alternative to SAW and quartz filters in RF applications.

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VII REFERENCES


