MECHANICAL/ELECTRICAL POWER-AWARE IMPEDANCE MATCHING FOR DESIGN OF CAPACITIVE VIBRATION ENERGY HARVESTERS

Dimitri Galayko[1], Philippe Basset[2]
[1]LIP6, University of Paris-VI, France, [2]University Paris-Est, ESYCOM/ESIEE, France

Abstract This study presents a systematic approach for power-aware design of an optimized capacitive vibration energy scavenger. The proposed method is based on the mechanical domain impedance analysis. First we investigated the mechanical properties (impedance) of the capacitive transducer/conditioning circuit block. We found that this impedance depends strongly on the energy state of the conditioning circuit. From this conclusion we derived a method allowing to find the conditioning circuit operating mode (voltages on the capacitors) maximizing the power yield for a given resonant MEMS transducer block, a given conditioning circuit and given external vibration parameters. This method was successfully applied on the system composed from a charge pump and an inductive flyback circuit. The study was validated by behavioral modeling in VHDL-AMS language environment.

Keywords: vibration energy harvesting, mechanical impedance, electromechanical transducer

1 Introduction

Mechanical energy harvesters using capacitive electromechanical transducers have recently been objects of extensive research activities. They belong to the class of multiphysical systems with strong coupling between the physical domains, non-linear and time variable. These properties make them very complex and at the same time, extremely interesting for a theoretical study. The goal of such a study is to provide a comprehensensive and analytic relation between the harvester system parameters and the operation characteristics (performances), so that an optimization of the energy yield would be possible.

So far, most analytical studies were based on simplifying assumptions concerning the transducer behaviour (e.g., Columb-Damped Resonant Generator, [1]), and considered separately mechanical and electrical domains. Such studies provide an insight into fundamental trends and trade-offs related with the harvester physics, but are unsufficient for a systematic design of real energy scanvenging systems.

In this study we propose a complete analysis of a *coupled* behaviour of the system "resonator-transducer-conditioning circuit" without making any simplification about the linearity of the building blocks. We apply this analysis to the electrostatic harvester which uses the conditioning circuit with architecture presented in [3], but the developed method is valid any other architectures as well.

In the proposed approach, the harvester is considered as a mechanical system composed from a source

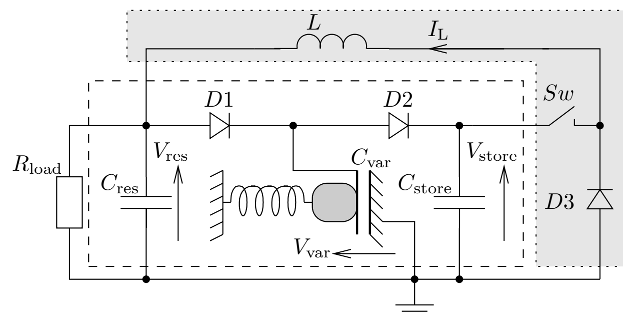

Figure 1: Electrical scheme of the harvester.

of an external force (the external vibrations), from a second-order lumped-parameter linear resonator and from a capacitive transducer associated with its conditioning circuit (fig. 1). For commodity, this mechanical system is analyzed through its electrical network (fig. 2). Here, the second newtonian law is modeled by the Kirschoff mesh law, the external force is modeled by a voltage source, the resonator is modeled by a reactive linear network with impedance Z_m. The transducer is modeled by a dipole generating some voltage (force) on its terminals. The force generated by the transducer depends on the resonator vibration amplitude and on the electrical state of the conditioning circuit.

The paper is organized in the following way. In the section 2 we present our approach to the mechanical analysis of the harvester. In the section 3 we present a coupled electromechanical analysis of the block "transducer/conditioning circuit". In the section 4 we show how a design of optimal harvester

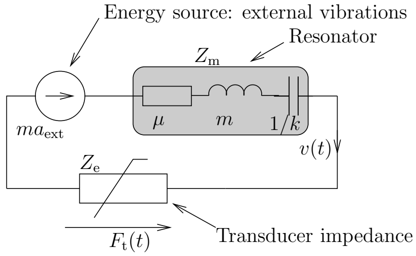

Figure 2: Equivalent electrical network of the harvester.

is possible. In the section 5 we present the modeling experiment validating the theory.

2 Mechanical analysis of harvester

Mechanical modeling of electrostatic harvesters has been presented in numerous papers [1]. Here we give some assumptions and definitions necessary for the further demonstrations.

1. The external vibrations are supposed to be sinusoidal.

2. The resonator is supposed to be narrow-band.

From 1) and 2), even if the transducer is non-linear, the mass vibrations are still close to sinusoidal, with the same frequency as the external vibrations.

3. We say that the transducer is non-linear, since :

– even if the mass displacement is sinusoidal, the generated transducer force is non-sinusoidal,

– the force generated by the transducer doesn't scale linearly with the amplitude of the mobile mass displacement.

4. We suppose that the transducer force is periodic and have the same fundamental frequency as the external vibrations.

Note that the assumption (4) is generally wrong, especially if the transducer is an active system. But in our, case, this assumption is correct since we intend to design a harvester which operates in a steady state corresponding to the maximal energy yield.

The power harvested by the transducer is given by the formula:

$$P = -\frac{1}{T}\int_T f_t v \mathrm{d}t, \qquad (1)$$

The velocity v is sinusoidal, the force f_t is periodic and non-sinusoidal, with the same period T: it can be proven that only the fundamental component of f_t contributes to P. Thus, considering only the fundamental sinusoidal components of all system quantities, it is possible to analyse the network through the formalism of complex amplitudes. This formalism uses extensively the notion of impedance. A mechanical impedance is defined as the ratio between the complex amplitude of force and velocity (the complex amplitudes are named by dotted capial letters):

$$Z_{mech} = \frac{\dot{F}}{\dot{V}}. \qquad (2)$$

This is the general definition of the mechanical impedance, and it is usually employed for linear systems. To apply this definition to a non-linear transducer, we add to this definition that $\dot{F}$ represents the complex amplitude of the fundamental sinusoidal component of the force (generated by the transducer) when the mass moves sinusoidly with a complex velocity $\dot{V}$.

The formalism of complex amplitude allows to replace the differential newtonian equation by a complex algebraic equation which is the mesh equation for the equivalent network (fig. 2):

$$m\dot{A}_{ext} = (Z_m + Z_t)\dot{V}, \qquad (3)$$

where $\dot{A}_{ext}$ is the complex amplitude of the external vibration acceleration, m is the mass, Z_m is the impedance of the resonator, Z_t is the impedance of the transducer. The phase of the external vibrations acceleration is taken to be zero, hence, $\dot{A}_{ext} = |\dot{A}_{ext}|$.

For the harvested power, (1) becomes:

$$P = 0.5|\dot{V}|^2 \mathrm{Re}\, Z_t. \qquad (4)$$

Hence, to maximize the power yield, both the amplitude of the mass vibration and the real part of the transducer impedance are to be maximized.

In the next section we provide an insight into the calculation of the transducer's impedance.

3 Electromechanical properties of the harvester

3.1 Electrical optimization of the conditioning circuit

The conditioning circuit presented in fig. 1 is composed from a charge pump and from a flyback circuit (in gray). The role of the chargepump is to transfer the charge from a large reservoir capacitor C_{res} toward a much smaller capacitor C_{store}, making use of the variation of the transducer capacitance (C_{var}). During the pumping, starting from a state where the voltages of the three capacitors are equal, the voltage V_{store} increases, V_{res} decreases very slightly (since $C_{res} \gg C_{store}$). After some number of pumping cycles, the V_{store} growth is saturated. Fig. 3 presents an example of evolution of V_{store} and the corresponding flow (power) of the harvested energy which is accumulated in the system of capacitors C_{store} and C_{res} connected in series. It can be seen that at saturation, the harvested power is close to zero.

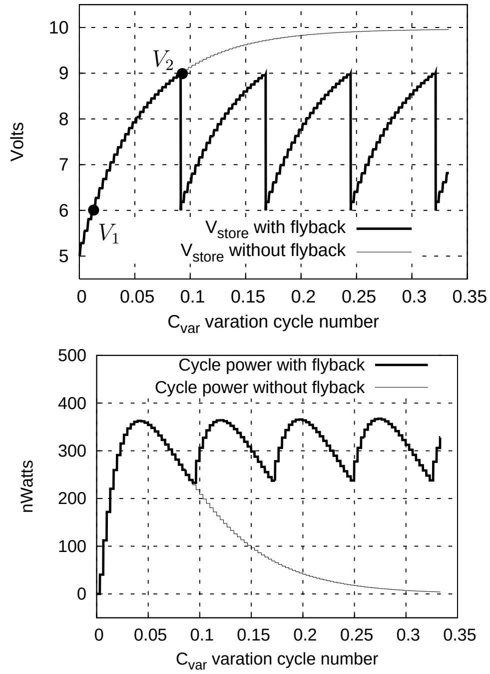

Figure 3: Example of operation of the conditioning circuit with and without flyback (thick and thin lines respectively). Top: V_{store} evolution, bottom: evolution of the average power harvested during one C_{var} variation cycle.

The flyback circuit is activated by the switch SW when the chargepump approaches the saturation. The flyback circuit has two roles. Firstly, it returns the charge pump to some earlier state away from the saturation, so to allow a continuous harvested energy flow. Secondly, it takes away the harvested energy from the system $C_{res}C_{store}$ and, using the inductive energy buffer L, puts it in the reservoir capacitor C_{res}, which is connected with the load supply circuit. This operation is presented by the thick line curves in fig. 3.

The plots of fig. 3 were obtained by modeling, under conditions that the resonator displacement amplitude and the C_{var} variation are constant. This plot helps to make a choice of the thresholds V_1 and V_2 corresponding to the switching instants. It can be seen that V_1 and V_2 points should be distributed around the maximum of the power curve of the lower plot. Theoretical calculation provides the value of the V_{store} at which the harvested power is maximal:

$$V_{store\ optimal} = V_{res} \frac{\gamma + \frac{\sigma}{\gamma} + 1}{\frac{\sigma}{\gamma} + 2}, \qquad (5)$$

where $\gamma = C_{max}/C_{min}$, $\sigma = C_{max}/C_{store}$.

3.2 Transducer impedance

To calculate the transducer impedance following the definition of section 2, it is necessary to calculate the time evolution of the real transducer force $f_t(t)$ *during one vibration period* and to expand it in complex Fourier series: $\dot{F}_t$ is equal to the first (fundamental) complex Fourier coefficient, and the impedance can be calculated using (2). Thus, the transducer's impedance is defined on one vibration period. Note that this definition doesn't require a periodicity of the transducer force.

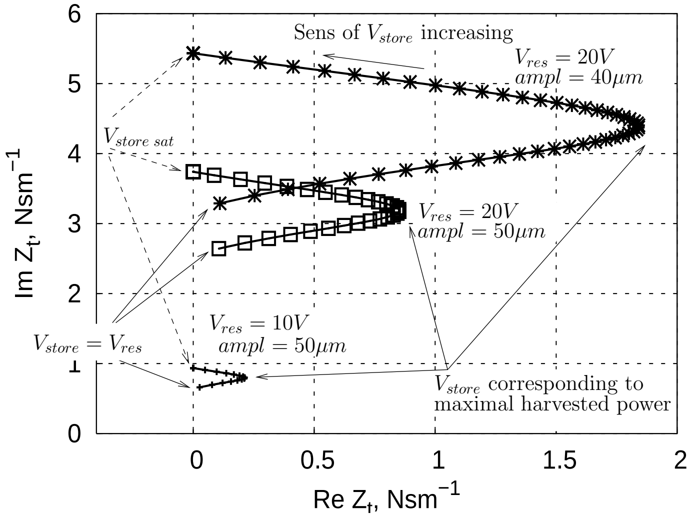

Figure 4: Evolution of the transducer impedance during the charge pump operation.

Fig. 4 presents three examples of the transducer impedance evolution with V_{store} during the chargepump operation, with different amplitudes and V_{res} voltages. It can be seen that the real part of the impedance has a maximum: this maximum happens exactly to the optimal V_{store} voltage given by the formula (5) obtained by the electrical analysis. As well, the formula (4) obtained by the mechanical analysis states that at this point of maximal ReZ_t the energy yield is maximal.

The transducer impedance depends on the mass vibration amplitude $|\dot{X}|$. The transducer impedance value can only be deduced numerically, even for the simpliest transducers with linear $C_{var}(x)$ relation.

The good news is that the function $Z_t(V_{res}, V_{store}, |\dot{X}|)$ scales quadratically with V_{res} and V_{store}, as can be noticed from the plots of the fig. 4 obtained with $|\dot{X}| = 50\mu m$. Thus, it can be tabulated for one value of V_{res}, for a given vibraiton amplitude.

4 Optimal harvester design

The optimisation presented in the precedent section assumes a constant and known vibration amplitude. However, from (3), the amplitude of the mass vibration depends on the mechanical impedance of the transducer, which depends itself on the vibration amplitude. Thus, the problem is formulated as follows: given a $C_{var}(x)$ of the transducer, given a resonator with impedance Z_m and given the amplitude of the external vibrations, find the optimal operation conditions of the resonator, i.e., the voltages V_{res} and V_{store} and the resonator vibration amplitude.

The optimization of the electrical part ties V_{res} and V_{store}: we always want $V_{store} = V_{store\ opt}$ for a given V_{res} and the resonator vibration amplitude. Thus, since $Z(V_{res}, V_{store}, |\dot{X}|)$ scales quadratically with V_{res} and V_{store}, we have:

$$Z(V_{res}, V_{store\ opt}, |\dot{X}|) = \quad (6)$$

$$Z(V_{res0}, V_{store\ opt0}, |\dot{X}|) \cdot \left(\frac{V_{res}}{V_{res\ 0}}\right)^2 = \alpha(|\dot{X}|)V_{res}^2, \quad (7)$$

where α is a complex number calculated numerically for a given amplitude and for an arbitrary V_{res0} value.

We now have one complex equation (3) and tree free real unknown parameters, V_{res} and the absolute value and the argument of the complex vibration amplitude. We propose to fix the needed mass vibration amplitude. This does make sense since to maximize the harvested power, the mass vibration amplitude should be as high as possible following the formula (4). Thus, in the most cases, the amplitude of the mass vibration should be chosen close to the maximal amplitude allowed by the system geometry.

After the amplitude is chosen, the equation (4) provides an unique solution for the absolute value of V_{res}:

$$V_{res\ opt}^2 = \frac{m|\dot{A}_{ext}|}{\omega|\dot{X}||\alpha|}\sqrt{1 - \left[\frac{|Z_m|\omega|\dot{X}|}{m|\dot{A}_{ext}|}sin(\phi_Z - \phi_\alpha)\right]^2} \quad (8)$$

$$-\frac{|Z_m|}{|\alpha|}cos(\phi_Z - \phi_\alpha), \quad (9)$$

where m is the transducer mass, ϕ_Z and ϕ_α are the arguments of the resonator impedance and the value $\alpha(|\dot{X}|)$.

This V_{res} guarantees that for the chosen vibration amplitude, the power yield is optimal.

5 Validation of the theory

To validate the theory, we designed a harvester with the parameters and $C_{var}(x)$ profile given in [2]. We chosen the mass vibration amplitude to be 47 μm: our algorithm highlighted that an optimal energy yield is obtained when $V_{res} = 8.7\ V$, $V_{store} = 20.8\ V$.

We modeled the harvester using a precise VHDL-AMS model [2]. The results agreed perfectly. An example of results of the modeling experiment is shown in fig 5: exactly 47 μm of the resonator displacement was obtained for the optimum value of V_{res},

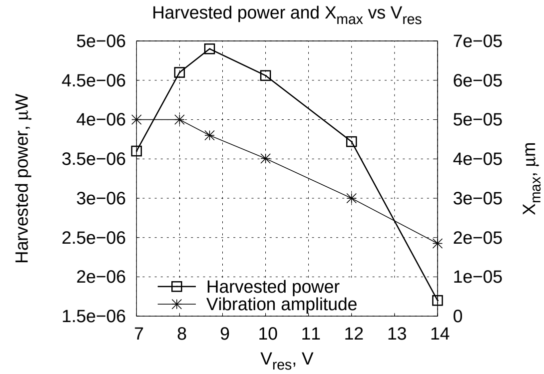

Figure 5: Modeling results: vibration amplitude and harvester power with $V_{store} = 20.8V$ ($V_1 = 20V$, $V_2 = 21.6V$).

which corresponds to a maximal harvested power of 4.9 μW, also predicted by the analysis. Moreover, making the experiment with V_{res} voltages different from the prescribed value of 8.7 V, we systematically observed degradation in the harvested power and, for lower V_{res}, an unwanted contact with the stoppers located at +/- 50 μm (fig. 5).

6 Conclusion and future work

The proposed approach of the harvester system analysis provides an insight into the mechanism defining the energy yield. As it can be seen, this technique is much more complex than a simple impedance matching approach: firstly, because of the non-linearity of the system, secondly, because of the structural limitations of the available transducer impedance range, which are imposed by the physics of the transducer and by the architecture of the conditioning circuit.

This study opens a way for a more deep investigation of the fundamental properties of the system. For example, given the non-linearity of the studied system, the question about the stability of the solution provided by the formula (9) remains open.

7 Acknowledgments

This work is partially funded by the French National Research Agency through the contract N° JC05-54551 and is supported by the French competitiveness cluster *AdvanCity*.

References

[1] P. D. Mitcheson at al., Journal of Microelectromechanical systems, vol. 13, no. 3, June 2004

[2] D. Galayko et al., BMAS conference, 2007

[3] B. C. Yen et al., IEEE Transactions on Circuits and Systems, vol. 53, february 2006