Single Stage Amplifiers

- Basic Concepts
- Common Source Stage
- Source Follower
- Common Gate Stage
- Cascode Stage

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References

- **B. Razavi, "Design of Analog CMOS Integrated Circuits", McGraw-Hill, 2001.**

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Basic Concepts I

- **Amplification is an essential function in most analog circuits !**

- **Why do we amplify a signal ?**

    - **The signal is too small to drive a load**
    - **To overcome the noise of a subsequent stage**
    - **Amplification plays a critical role in feedback systems**

In this lecture:

- **Low frequency behavior of single stage CMOS amplifiers:**
    - **Common Source, Common Gate, Source Follower, ...**
- **Large and small signal analysis.**
- **We begin with a simple model and gradually add 2nd order effects**

⇨ **Understand basic building blocks for more complex systems.**

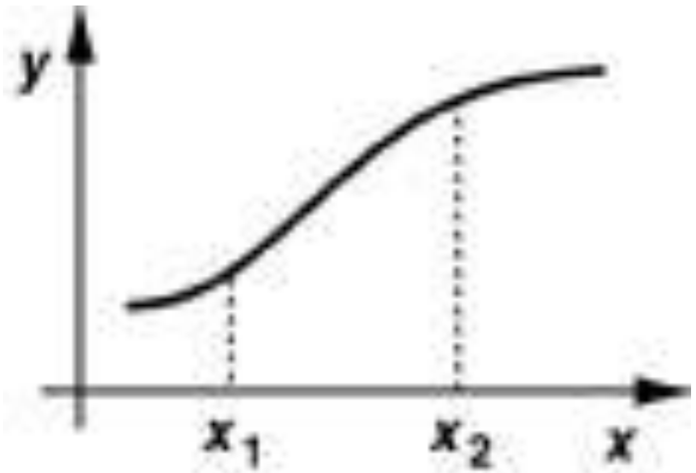
Approximation of a nonlinear system

Input-Output Characteristic of a nonlinear system

$$y(t) \approx \alpha_0 + \alpha_1 x(t) + \alpha_2 x^2(t) + \ldots + \alpha_n x^n(t) \qquad x_1 \le x \le x_2$$

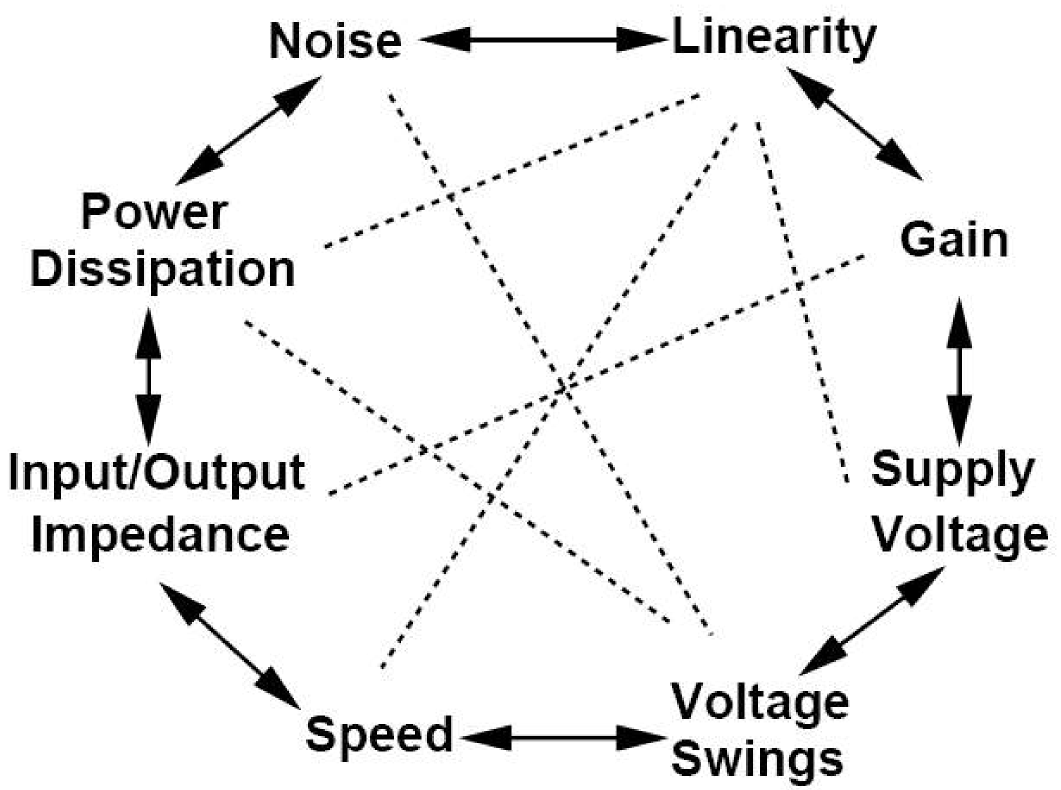
In a sufficiently narrow range:

$$y(t) \approx \alpha_0 + \alpha_1 x(t)$$



where α_0 can be considered the operating (bias) point and α_1 the small signal gain

Analog Design Octagon



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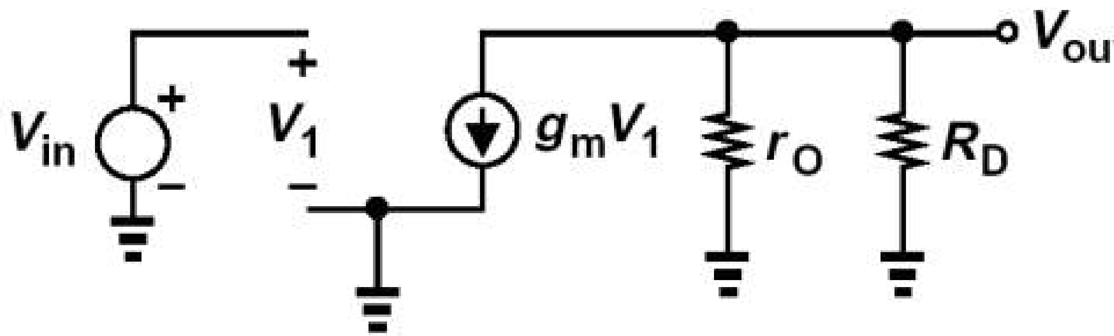
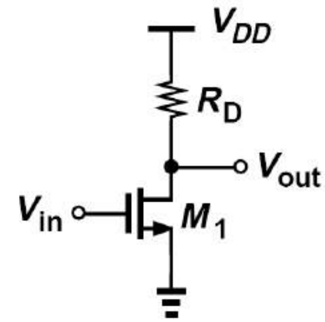
Taking Channel Length Modulation into account

Calculating A_v starting from the Small Signal model:





(a)

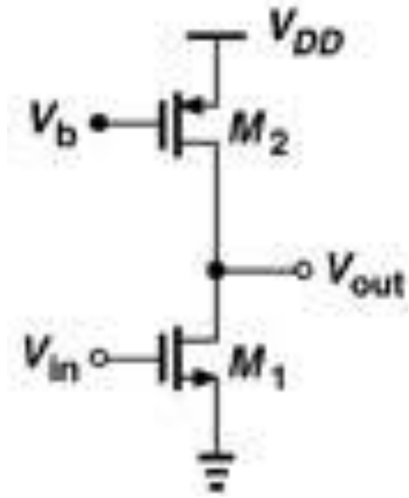
$$g_m V_1 (r_O // R_D) = -V_{out}$$

$$V_1 = V_{in}$$

$$A_v = \frac{V_{out}}{V_{in}} = -g_m (r_O // R_D)$$

CS Stage with Current-Source Load

- **Both transistors operate in the saturation region:**



$$A_v = -g_m (r_{O1} // r_{O2})$$

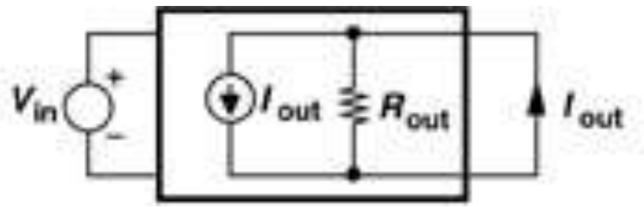
General expression to calculate A_v by inspection

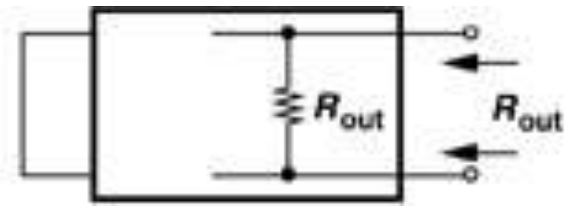
Lemma:

$$Av = -G_m R_{out}$$

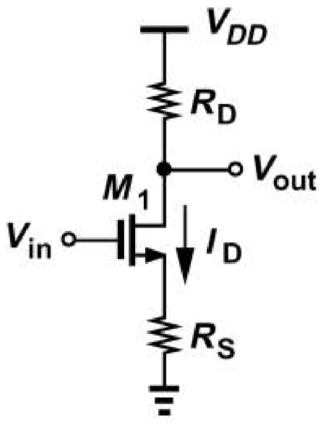
G_m **: the transconductance of the circuit when the output is shorted to grounded.**

R_{out} **: the output resistance of the circuit when the input voltage is set to zero.**



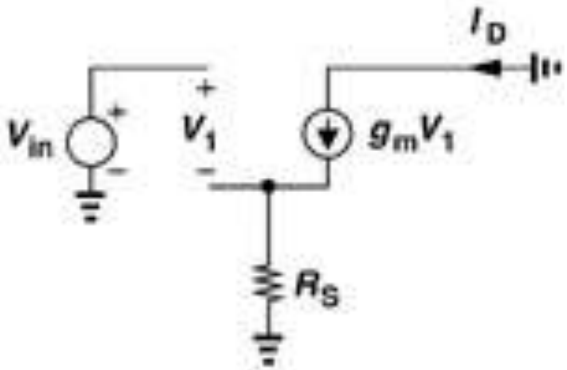


CS with Source Degeneration



(a)

Small Signal model:



(b)

$$G_m = \frac{I_D}{V_{in}} = \frac{g_m V_1}{V_1 + g_m V_1 R_S}$$

$$G_m = \frac{g_m}{1 + g_m R_S}$$

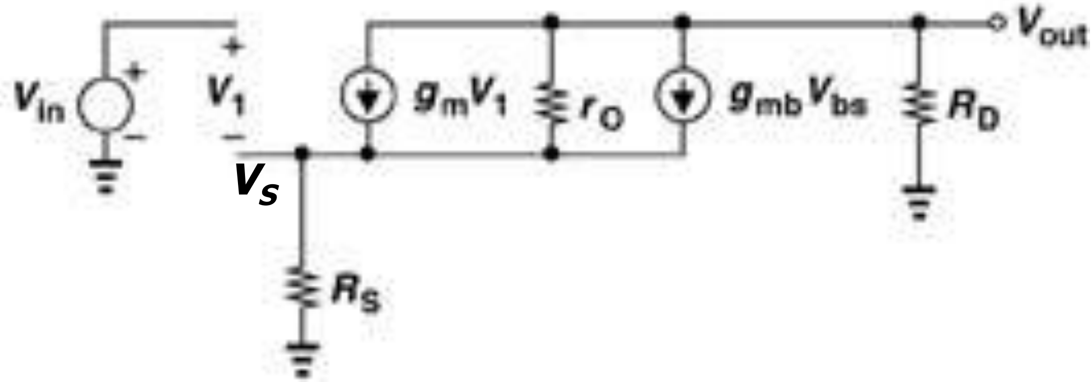
$$A_v = -G_m R_D \quad \rightarrow \quad A_v = -\frac{g_m R_D}{1 + g_m R_S}$$

Voltage Gain of Degenerated CS

Small Signal model including body effect & channel length modulation:

$$I_{R_D} = I_{R_S} = \frac{-V_{out}}{R_D}$$

$$\Rightarrow \quad V_S = -V_{out}\frac{R_S}{R_D}$$



The current through r_O : $$I_{r_O} = -\frac{V_{out}}{R_D} - (g_m V_1 + g_{mb} V_{BS})$$

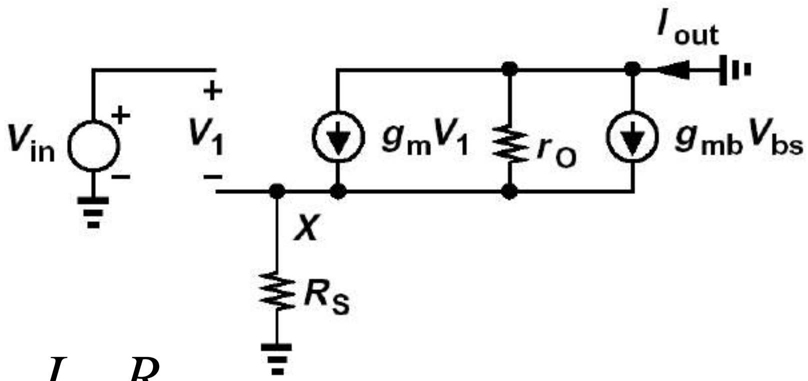
$$I_{r_O} = -\frac{V_{out}}{R_D} - [g_m(V_{in} + V_{out}\frac{R_S}{R_D}) + g_{mb} V_{out}\frac{R_S}{R_D}] \quad \Rightarrow \quad V_{out} = I_{r_O} r_O - \frac{V_{out}}{R_D} R_S$$

$$V_{out} = -\frac{V_{out}}{R_D} r_O - [g_m(V_{in} + V_{out}\frac{R_S}{R_D}) + g_{mb} V_{out}\frac{R_S}{R_D}] r_O - V_{out}\frac{R_S}{R_D}$$

$$\boxed{\frac{V_{out}}{V_{in}} = -\frac{g_m r_O R_D}{R_D + R_S + r_O + (g_m + g_{mb}) R_S r_O}}$$

Gm of Degenerated CS

Small Signal model including body effect and channel length modulation:



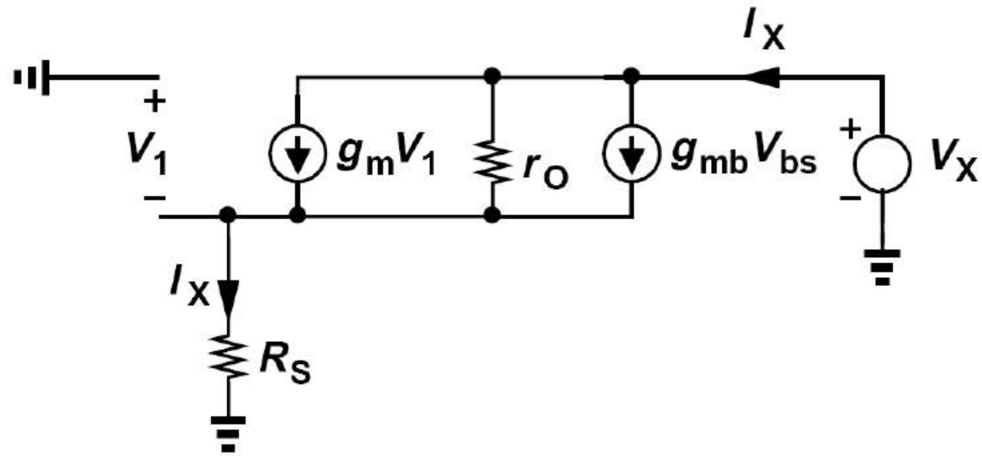
$$I_{out} = g_m V_1 - g_{mb} V_X - \frac{V_X}{r_O}$$

$$= g_m (V_{in} - I_{out} R_S) + g_{mb}(-I_{out} R_S) - \frac{I_{out} R_S}{r_O}$$

$$G_m = \frac{I_{out}}{V_{in}} = \frac{g_m r_O}{R_S + [1 + (g_m + g_{mb}) R_S] r_O}$$

Output Resistance of Degenerated CS

$$V_1 = -I_X R_S$$



The current flowing in r_O:

$$I_X - (g_m + g_{mb})V_1$$
$$= I_X + (g_m + g_{mb})R_S I_X$$

$$\Rightarrow \quad V_X = r_O[I_X + (g_m + g_{mb})R_S I_X] + I_X R_S$$

$$R_{out} = \frac{V_X}{I_X} = r_O[1 + (g_m + g_{mb})R_S] + R_S$$

$$R_{out} = [1 + (g_m + g_{mb})r_O]R_S + r_O$$

$$R_{out} \approx (g_m + g_{mb})r_O R_S + r_O \quad \Rightarrow \quad \boxed{R_{out} = [1 + (g_m + g_{mb})R_S]r_O}$$

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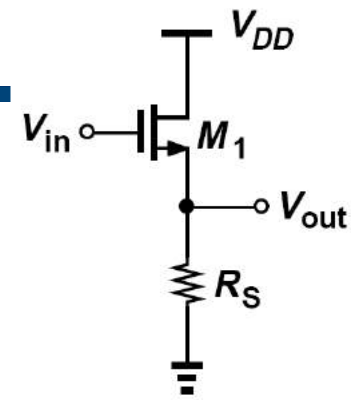
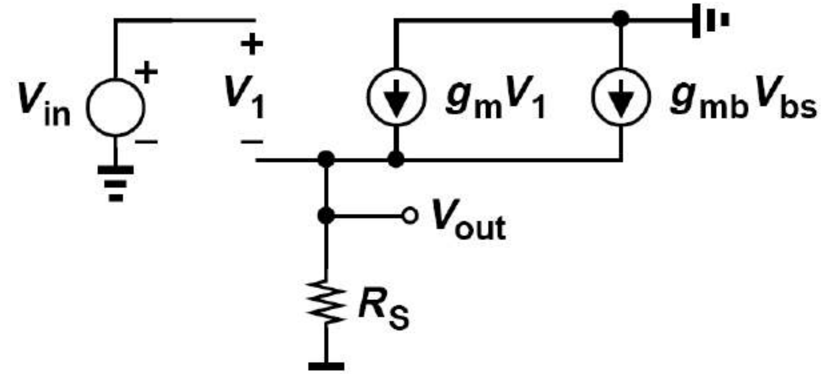
Source Follower Voltage Gain



(a)

$$V_{out} = \left[g_m V_1 + g_{mb} V_{BS}\right] R_S$$
$$= \left[g_m (V_{in} - V_{out}) - g_{mb} V_{out}\right] R_S$$

Small Signal Equivalent Circuit

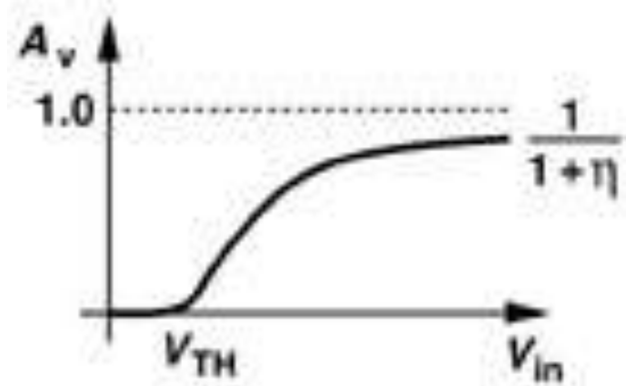


$$\Rightarrow A_v = \frac{V_{out}}{V_{in}} = \frac{g_m R_S}{1 + (g_m + g_{mb}) R_S}$$

Since: $g_{mb} = \eta \, g_m$

And for : $g_m R_S >> 1$

$$\Rightarrow \boxed{A_v \approx \frac{1}{(1+\eta)}}$$



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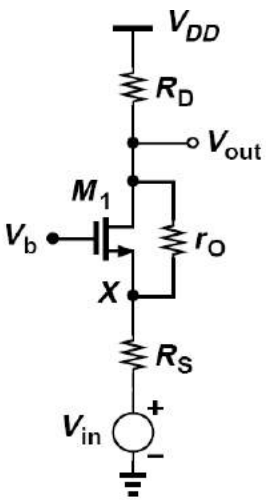
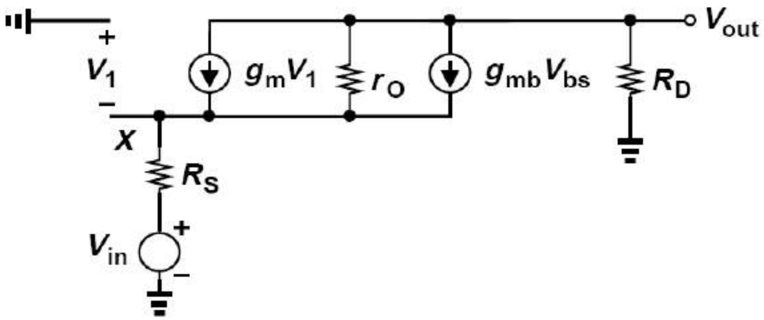
Common Gate Gain



Small Signal Signal Equivalent Circuit



The current through R_S is equal to $-V_{out}/R_D$:

$$V_1 - \frac{V_{out}}{R_D} R_S + V_{in} = 0$$

The current through r_O is equal to $-V_{out}/R_D - g_m V_1 - g_{mb} V_1$:

$$r_O\left(\frac{-V_{out}}{R_D} - g_m V_1 - g_{mb} V_1\right) - \frac{V_{out}}{R_D} R_S + V_{in} = V_{out}$$

$$r_O\left[\frac{-V_{out}}{R_D} - (g_m + g_{mb})\left(V_{out}\frac{R_S}{R_D} - V_{in}\right)\right] - \frac{V_{out}}{R_D} R_S + V_{in} = V_{out}$$

Common Gate Gain

Common Gate Amplifier:

$$A_{vCG} = \frac{(g_m + g_{mb})r_O + 1}{R_D + R_S + r_O + (g_m + g_{mb})r_O R_S} R_D$$

Degenerated Common Source Amplifier:

$$A_{vCS} = -\frac{g_m r_O}{R_D + R_S + r_O + (g_m + g_{mb})r_O R_S} R_D$$

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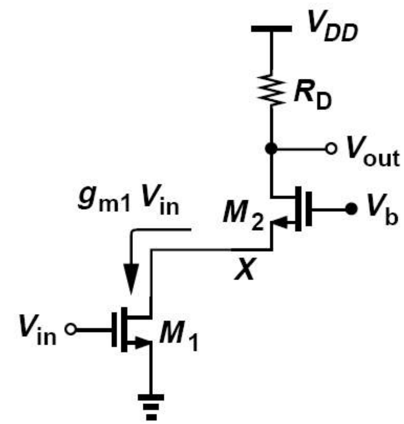
Biasing of a Cascode Stage

The cascade of CS stage and a CG stage is called "cascode".

M1 : the input device
M2 : the cascode device



Biasing conditions:

- **M1 in saturation:**

$$V_X = V_b - V_{GS2}$$

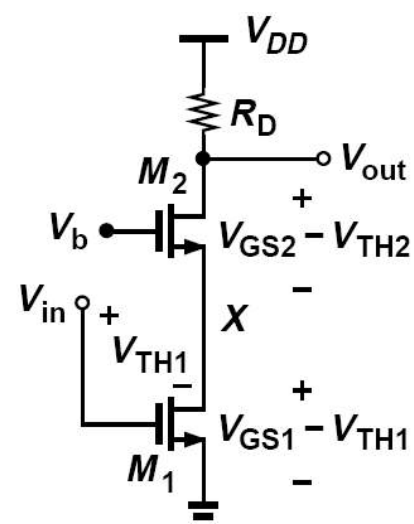
$$V_b - V_{GS2} \geq V_{in} - V_{TH1}$$

$$V_b \geq V_{in} + V_{GS2} - V_{TH1}$$

- **M2 in saturation:**

$$V_{out} - V_X \geq V_b - V_X - V_{TH2}$$

$$\boxed{V_{out} \geq V_{in} - V_{TH1} + V_{GS2} - V_{TH2}}$$



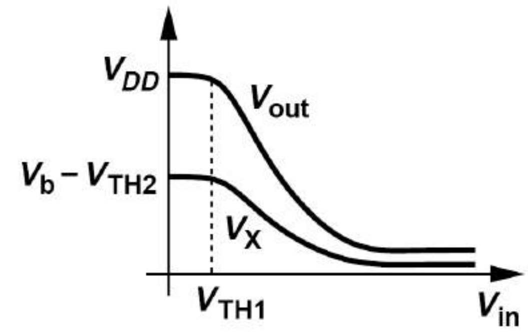
Cascode Stage Characteristics

Large signal behavior:

As V_{in} goes from zero to V_{DD}

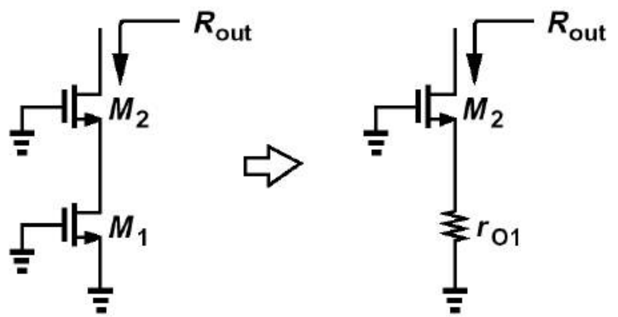
For $V_{in} < V_{TH}$ M1 and M2 are OFF

➡ $V_{out} = V_{DD}$



Output Resistance:

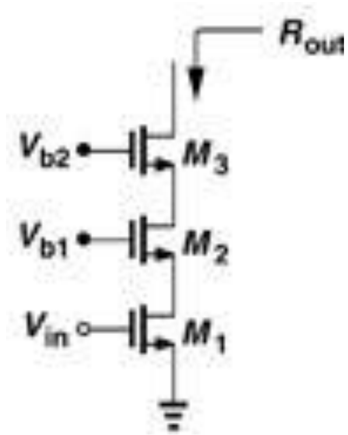
- **Same common source stage with a degeneration resistor equal to r_{O1}**



$$R_{out} = [1 + (g_{m2} + g_{mb2})r_{O2}]r_{O1} + r_{O2}$$

$$R_{out} \approx (g_{m2} + g_{mb2})r_{O2}r_{O1}$$

- **M2 boosts the output impedance of M1 by a factor of $g_m r_{02}$**



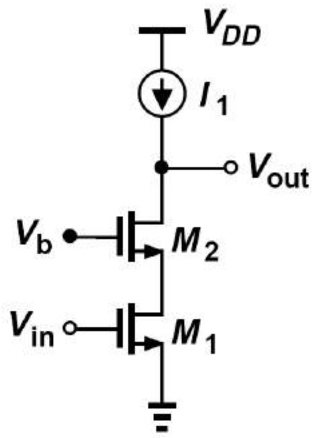
- **Triple cascode** $R_{out} \uparrow\uparrow$

➡ **difficult biasing at low supply voltage.**

Cascode Stage Voltage Gain

$$Av = -G_m R_{out}$$

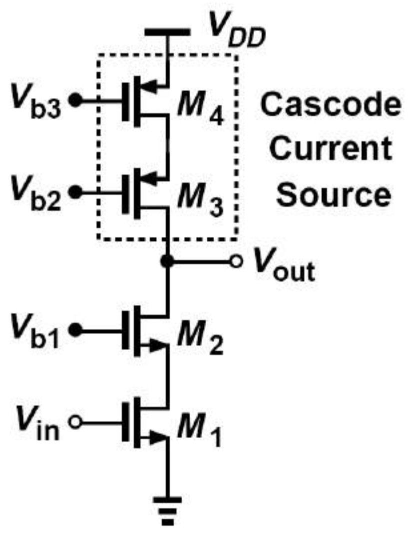
$$G_m \approx g_{m1}$$



Ideal Current Source:

$$R_{out} \approx (g_{m2} + g_{mb2}) r_{O2} r_{O1}$$

$$\boxed{A_v \approx (g_{m2} + g_{mb2}) r_{O2} \; g_{m1} r_{O1}}$$

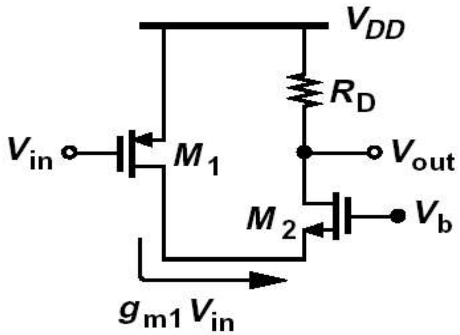


Cascode Current Source:

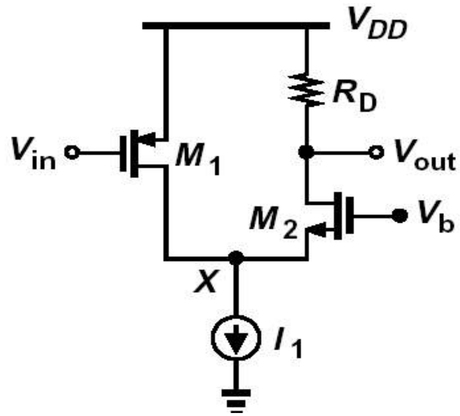
$$R_{out} \approx g_{m2} r_{O2} r_{O1} \,//\, g_{m3} r_{O3} r_{O4}$$

$$\boxed{A_v \approx g_{m1} \left(g_{m2} r_{O2} r_{O1} \,//\, g_{m3} r_{O3} r_{O4} \right)}$$

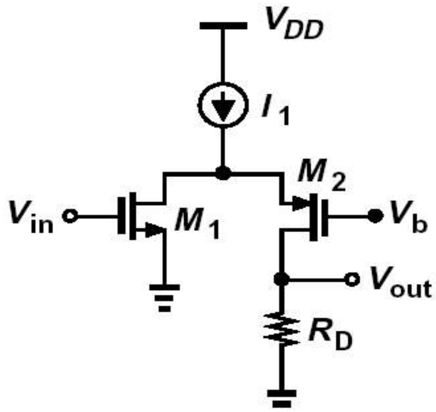
Folded Cascode



Simple Folded Cascode



Folded Cascode with biasing

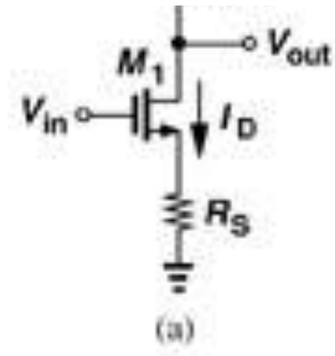


Folded Cascode with NMOS input

Output Resistance of Folded Cascode

Degenerated Common Source Stage:

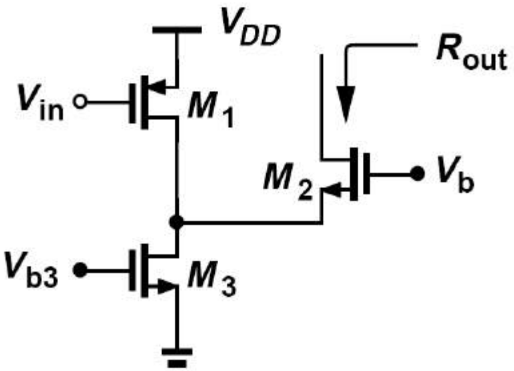
$$R_{out} = [1 + (g_{m1} + g_{mb1})r_{O1}]R_S + r_{O1}$$



Folded Cascode Stage:

M_1 ⇨ M_2

R_S ⇨ $r_{O1} // r_{O3}$



$$R_{out} = [1 + (g_{m2} + g_{mb2})r_{O2}](r_{O1} // r_{O3}) + r_{O2}$$

Voltage Gain of Degenerated CS

$$\frac{V_{out}}{V_{in}} = -\frac{g_m r_O R_D}{R_D + R_S + r_O + (g_m + g_{mb}) R_S r_O}$$

$$\frac{V_{out}}{V_{in}} = -\frac{g_m r_O}{R_S + [1 + (g_m + g_{mb}) R_S] r_O} \frac{R_D [R_S + r_O + (g_m + g_{mb}) R_S r_O]}{R_D + R_S + r_O + (g_m + g_{mb}) R_S r_O}$$

$$\frac{V_{out}}{V_{in}} = G_m \left(R_{out} // R_D \right)$$

The output resistance of a degenerated CS stage:

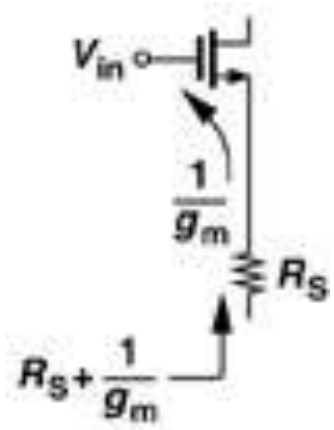
$$R_{out} = [1 + (g_m + g_{mb}) R_S] r_O$$

The Transconductance of a degenerated CS stage:

$$G_m = \frac{I_{out}}{V_{in}} = \frac{g_m r_O}{R_S + [1 + (g_m + g_{mb}) R_S] r_O}$$

Estimating Gain by Inspection

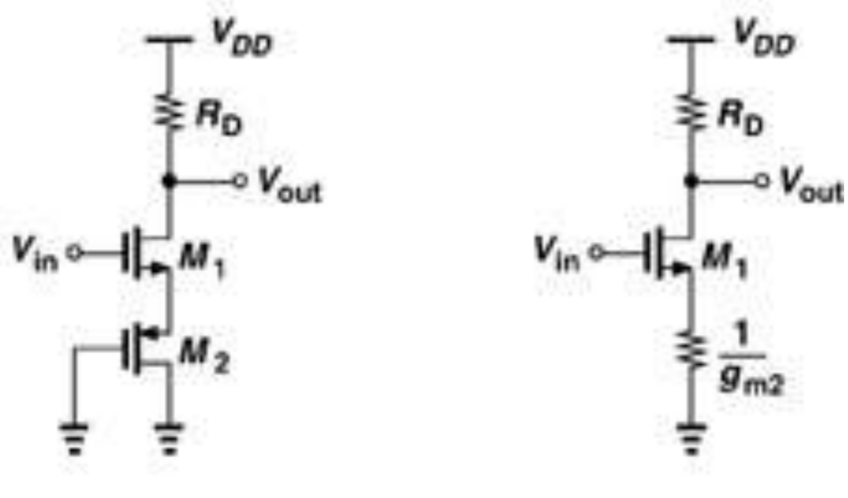
$$A_v = -\frac{g_m R_D}{1 + g_m R_S} = -\frac{R_D}{1/g_m + R_S}$$



$$Gain = -\frac{\text{Resistance seen at the Drain}}{\text{Total Resistance in the Source Path}}$$

Example:

$$A_v = -\frac{R_D}{1/g_{m1} + 1/g_{m2}}$$



(a) (b)

Differential Amplifiers

- ***Single Ended and Differential Operation***
- ***Basic Differential Pair***
- ***Common-Mode Response***
- ***Differential Pair with MOS loads***

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References

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- ***Single Ended and Differential Operation***
- ***Basic Differential Pair***
- ***Common-Mode Response***
- ***Differential Pair with MOS loads***

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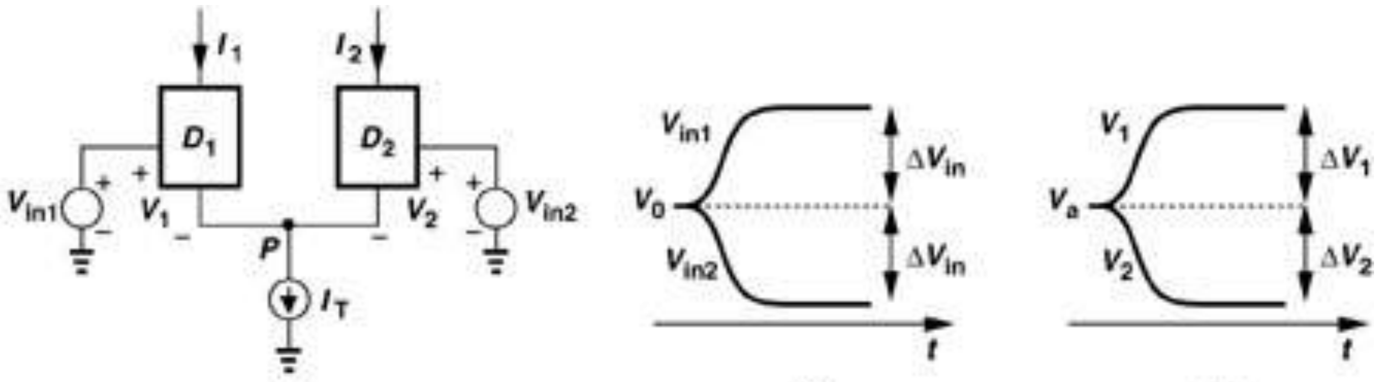
The concept of Half Circuit

If a fully symmetric differential pair senses differential inputs then the concept of half circuit can be applied.

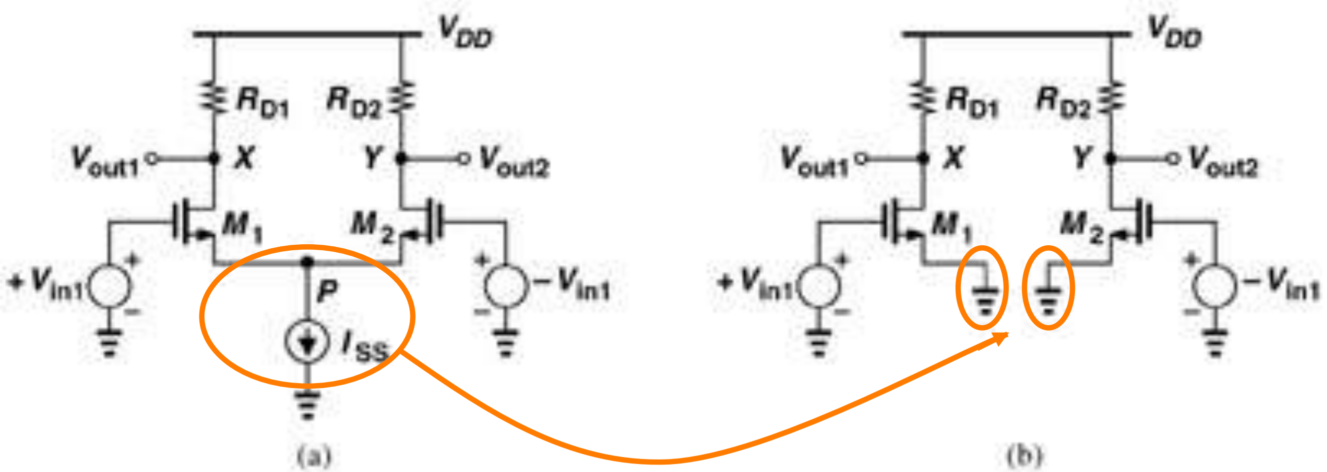


(a) (b) (c)

- ***A differential change in the inputs V_{in1} and V_{in2} is absorbed by V_1 and V_2 leaving V_P constant***

Application of The Half Circuit Concept

Since V_P experiences no change, node P can be considered "ac ground" and the circuit can be decomposed into two separate halves



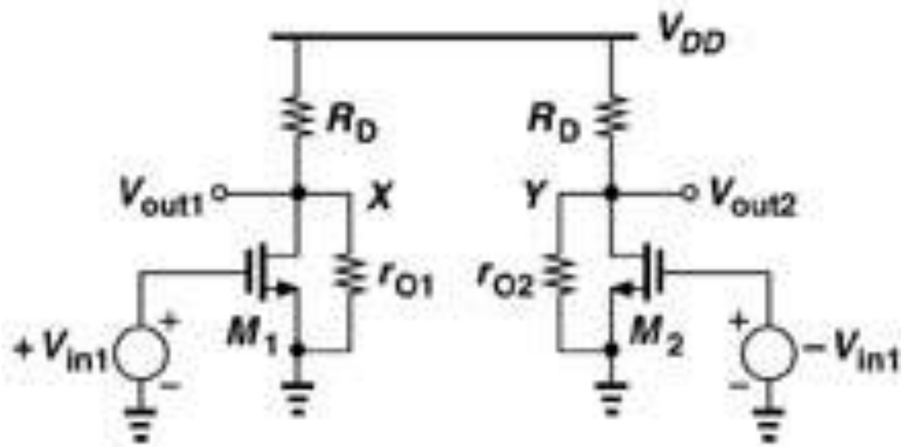
Two common source amplifiers:

$$\frac{V_X}{V_{in1}} = -g_m R_D \qquad \frac{V_Y}{V_{in2}} = -g_m R_D$$

$$\boxed{\frac{V_X - V_Y}{V_{in1} - V_{in2}} = -g_m R_D}$$

The Half Circuit Concept : Example

Taking into account the output resistance (channel length modulation)

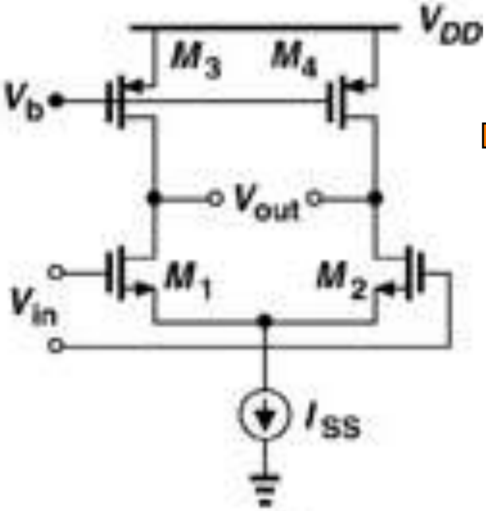


Two common source amplifiers:

$$\frac{V_X}{V_{in1}} = -g_m\left(R_D // r_{O1}\right) \qquad \frac{V_Y}{V_{in2}} = -g_m\left(R_D // r_{O2}\right)$$

$$\boxed{\frac{V_X - V_Y}{V_{in1} - V_{in2}} = -g_m\left(R_D // r_O\right)}$$

Cascode Differential Pair



(b)

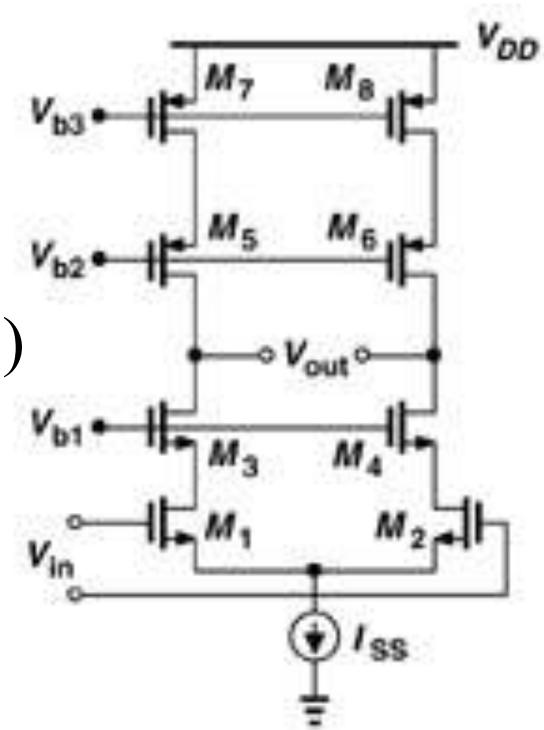
Current Source Load:

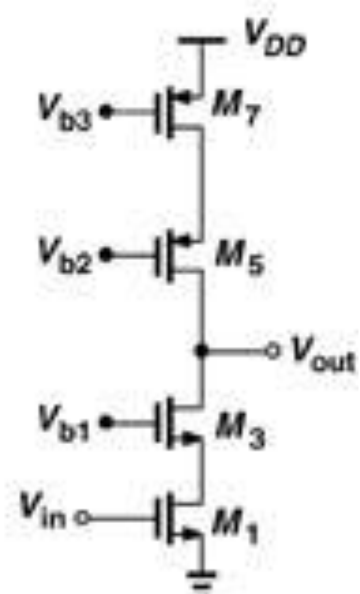
$$A_v = g_{mN}(r_{ON} // r_{OP})$$

Low gain 10 to 20.

To increase the gain:
Cascode Differential Pair

$$A_v = g_{m1}(g_{m3}r_{O3}r_{O1} // g_{m5}r_{O5}r_{O7})$$





Operational Amplifiers

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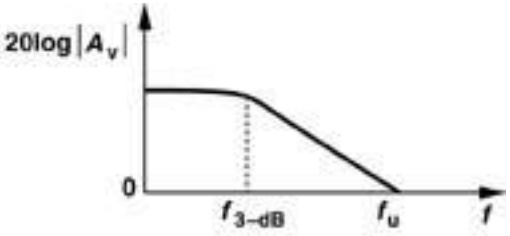
Operational Amplifier: Performance Parameters

Gain: the open loop gain of an op-amp determines the precision of the feedback system employing the op-amp



Small Signal Bandwith:
Unity-Gain freq., f_u, and the 3dB freq., $f_{3\text{-}dB}$.

Large Signal Bandwidth (slew rate):
Op-Amp response to large transient signals.

Output Swing:

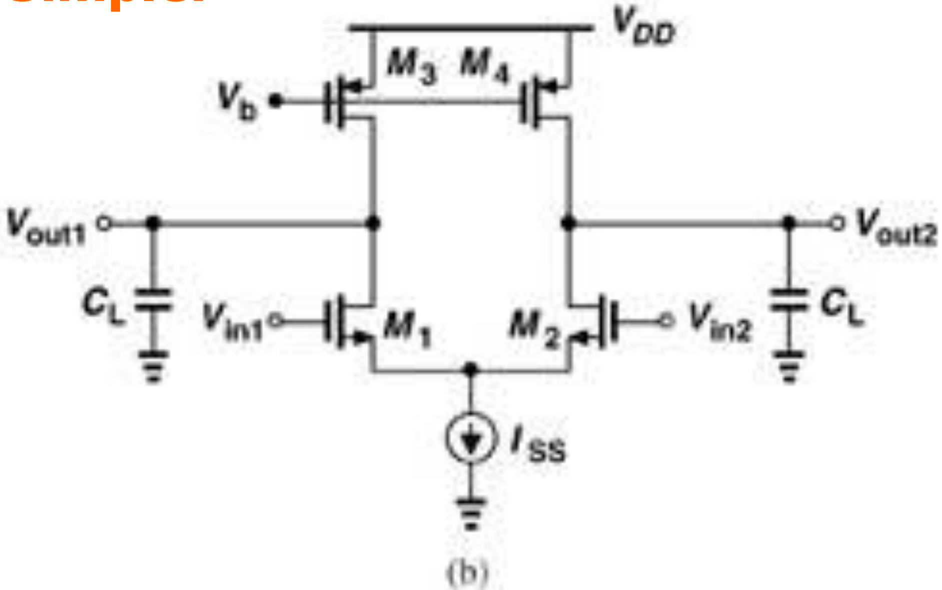
Linearity: non-linearity can be reduced by using a differential circuit and by increasing the open-loop gain in a feedback system

Noise and Offset: input noise and offset determine the minimum signal level that can be processed with reasonable quality.

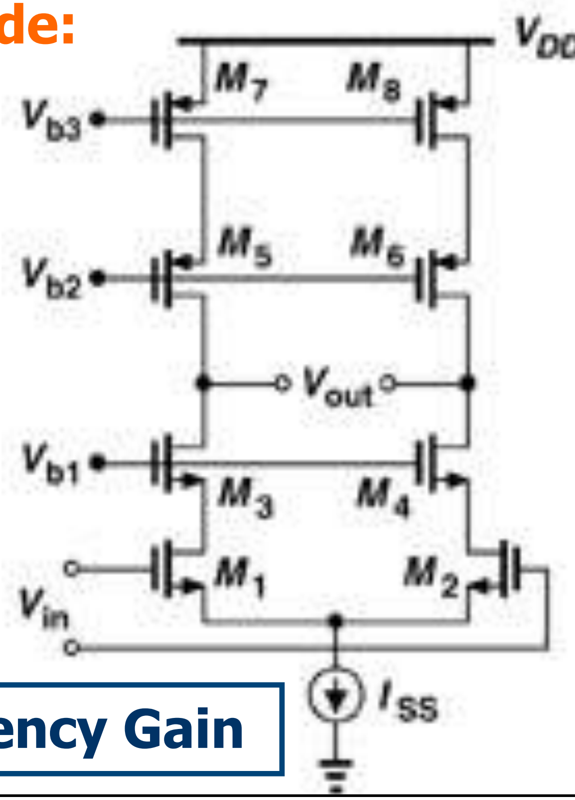
Supply Rejection:

Single stage Op-Amps

Simple:



Cascode:



Small Signal Low Frequency Gain

$$A_0 = g_{mN}(r_{ON} // r_{OP})$$

$$A_0 \leq 20$$

$$A_0 = g_{mN}[(g_{mN}r_{ON}^2 // g_{mP}r_{OP}^2)$$

Output Voltage Swing

$$2[V_{DD} - (V_{EG1} + V_{EG2} + V_{EGSS})]$$

$$2[V_{DD} - (V_{EG1} + V_{EG3} + V_{EG5} + V_{EG7} + V_{EGSS})]$$

Example:

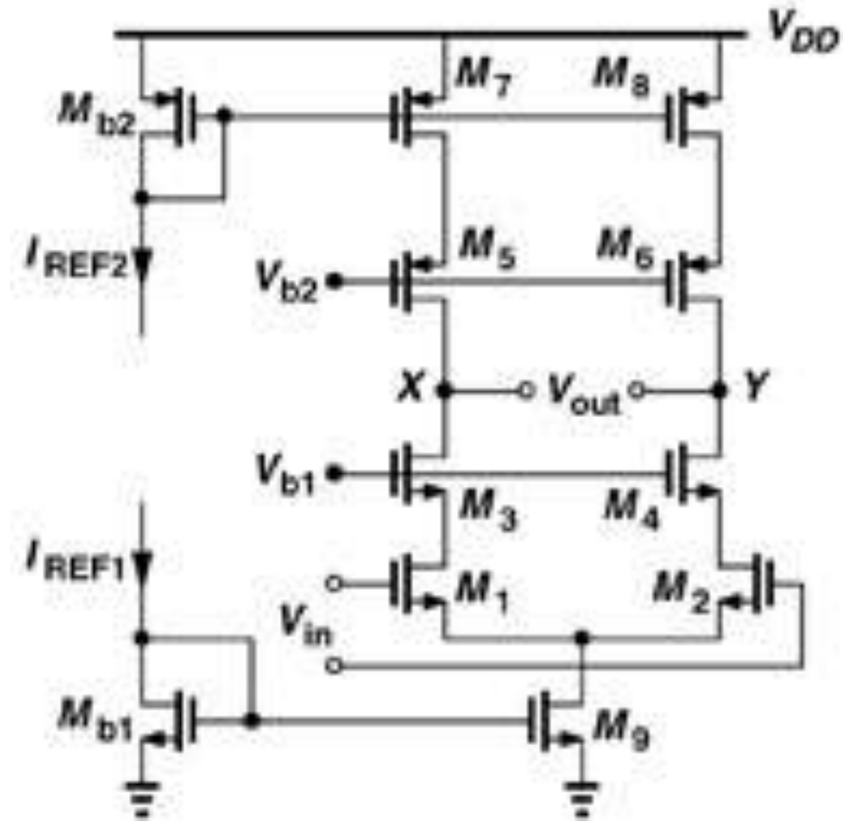
Design this amplifier (find all W/L as well as Vb1, Vb2 and Iref) with the following specifications:

$$V_{DD} = 3V$$

$$\text{Differential Output Swing} = 3V$$

$$\text{Power Dissipation} = 10mW$$

$$A_0 = 2000$$



Assume:

$$\mu_n C_{ox} = 60\,\mu A / V^2$$

$$\mu_p C_{ox} = 30\,\mu A / V^2$$

$$\lambda_n = 0.1\,V^{-1} \quad , \quad \lambda_p = 0.2\,V^{-1}$$

$$L_{eff} = 0.5\,\mu m$$

$$\gamma = 0 \quad , \quad V_{THN} = |V_{THP}| = 0.7V$$

Folded Cascode Circuits

The Idea:

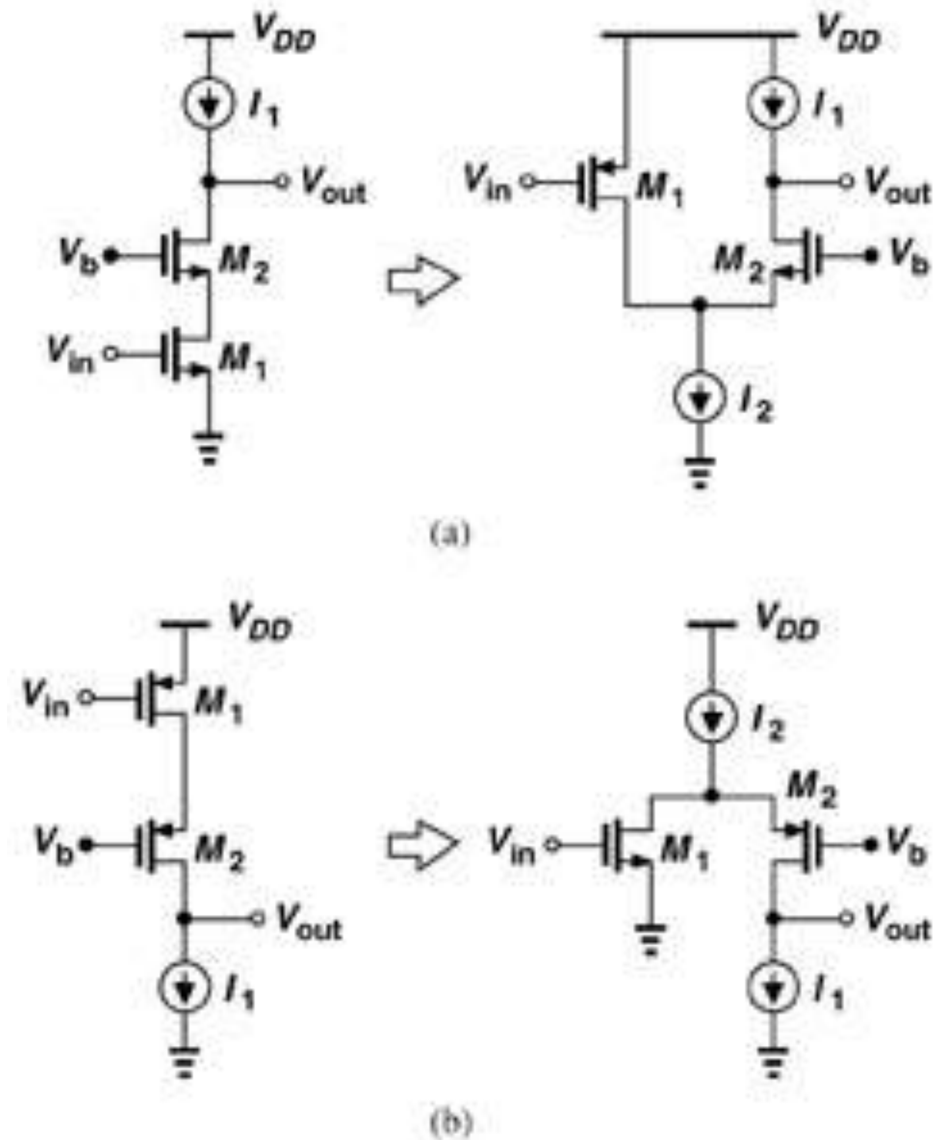
The input device is replaced by the opposite type.

Same Gain:

$$V_{out} = g_{m1} R_{out} V_{in}$$

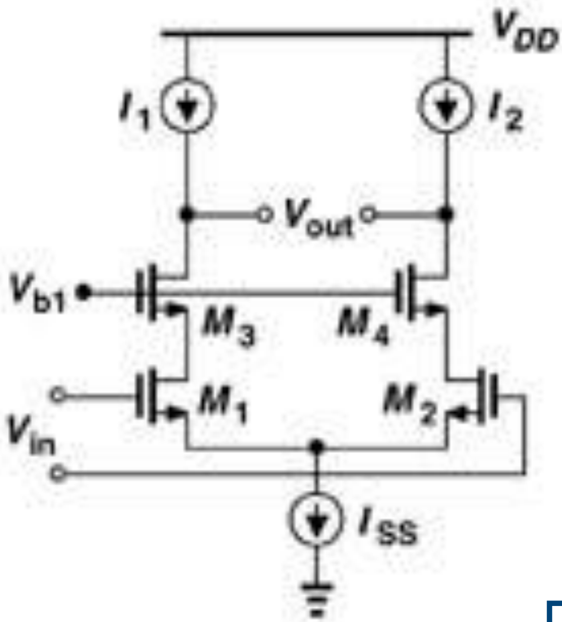
Advantage:

More room to choose the different voltage levels.



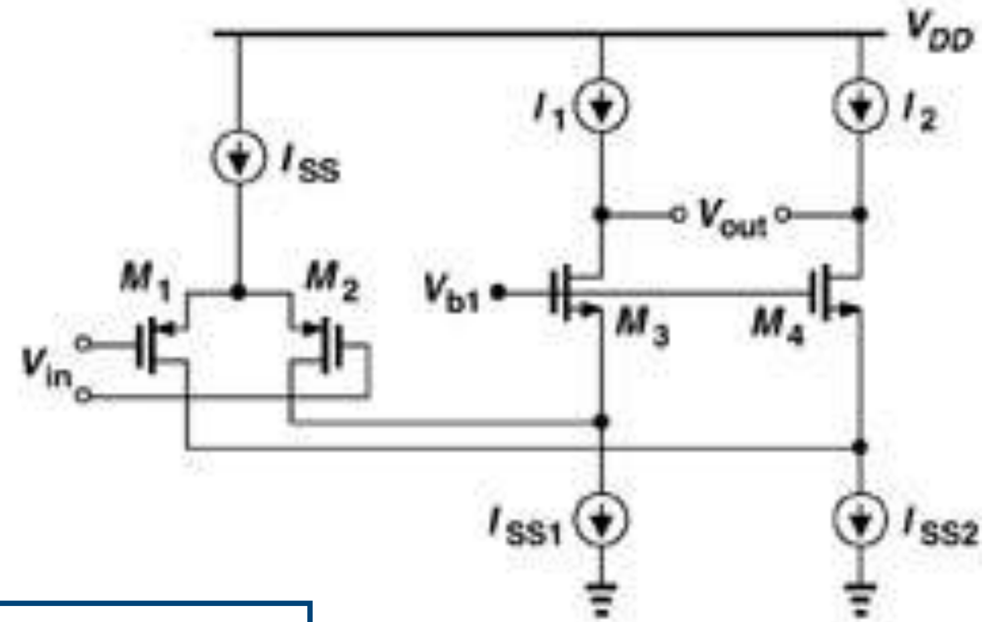
Folded Cascode Amplifier

Cascode



(a)

Folded Cascode



(b)

Biasing Current

Cascode:

$$I_{SS}$$

Folded Cascode:

$$I_{SS1} = \frac{I_{SS}}{2} + I_1$$

Input Common-Mode

Cascode:

$$V_{in,CM} < V_{b1} - V_{GS3} + V_{TH1}$$

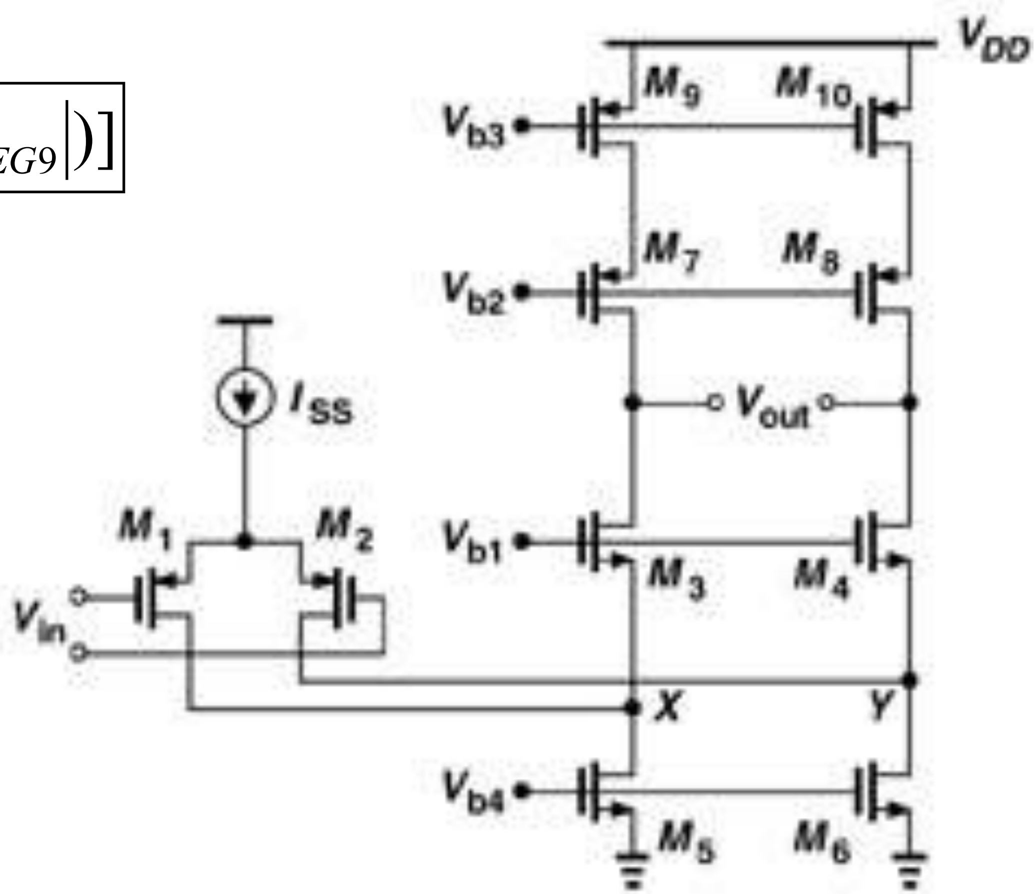
Folded Cascode:

$$V_{in,CM} > V_{b1} - V_{GS3} + \left|V_{TH1}\right|$$

Folded Cascode Amplifier

Output Voltage Swing

$$2[V_{DD} - (V_{EG3} + V_{EG5} + |V_{EG7}| + |V_{EG9}|)]$$

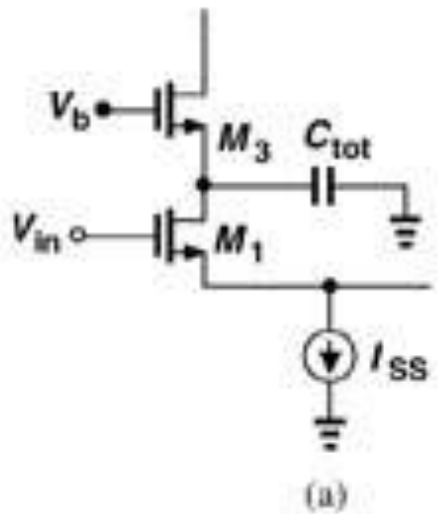


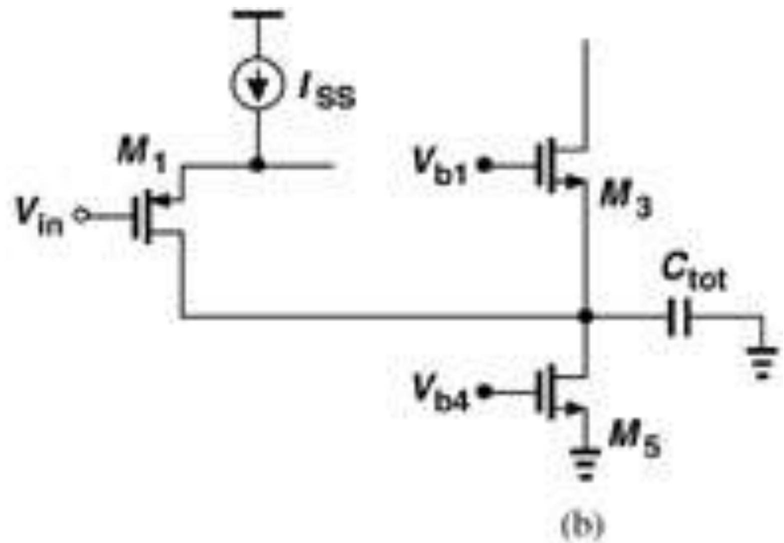
Small Signal Gain

$$A_0 \approx g_{m1}[g_{m3}r_{O3}(r_{O1} // r_{O5}) // g_{m7}r_{O7}r_{O9}]$$

Folded Cascode Amplifier

Telescopic | **Folded Cascode**



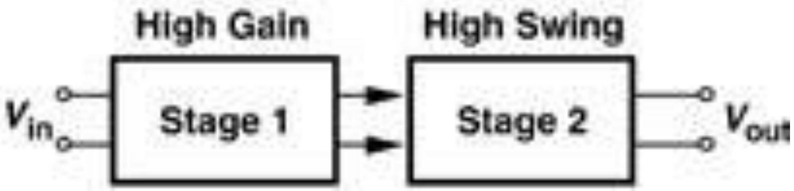


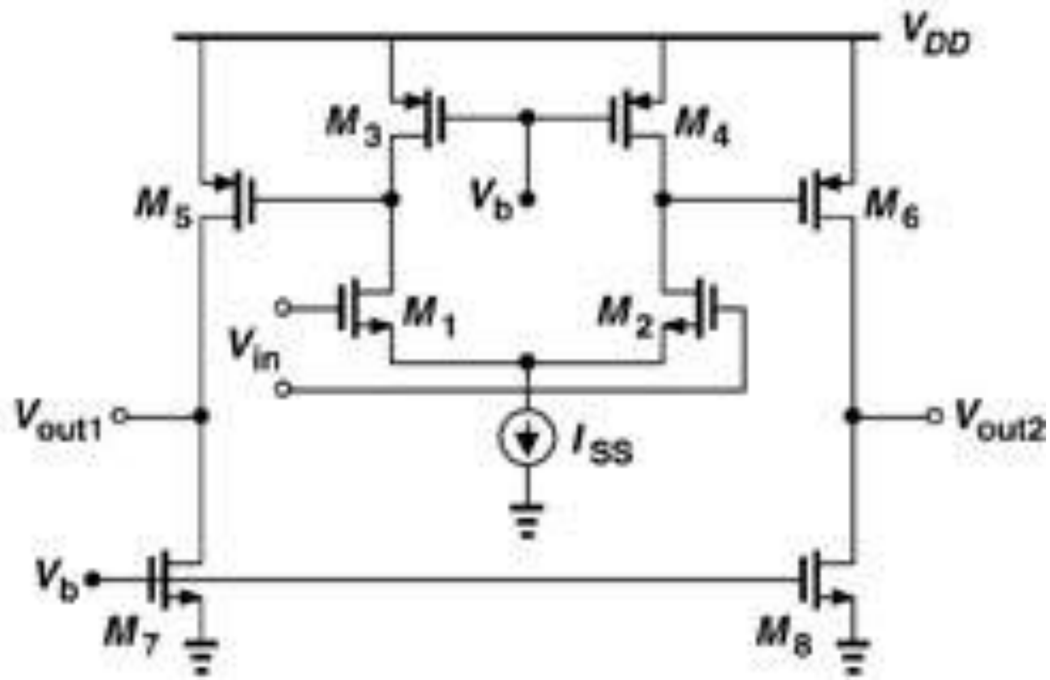
Effect of Device capacitance on the nondominant pole in telescopic and folded cascode

$$C_{tot} = C_{GS3} + C_{SB3} + C_{DB1} + C_{GD1}$$

$$C_{tot} = C_{GS3} + C_{SB3} + C_{DB1} + C_{GD1} + C_{GD5} + C_{DB5}$$

Two-Stage OpAmp





Output Voltage Swing

$$2[V_{DD} - (|V_{EG5}| + V_{EG7})]$$

Small Signal Gain

$$A_0 \approx g_{m1}(r_{O1} // r_{O3}) \times g_{m5}(r_{O5} // r_{O7})$$

Frequency Response of Amplifiers

- General Considerations
  - Miller Effect
  - Association of Poles with Nodes
- Common Source Stage
- Source Follower
- Differential Pair

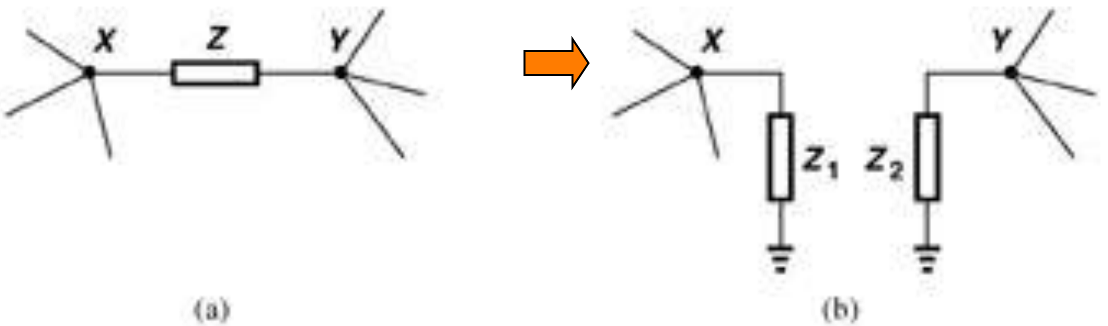
Hassan Aboushady
University of Paris VI

References

- B. Razavi, “Design of Analog CMOS Integrated Circuits”, McGraw-Hill, 2001.

Miller Effect

- **Miller's Theorem**



with $A_v = \frac{V_Y}{V_X}$ **we have** $Z_1 = \frac{Z}{1 - A_v}$ $Z_2 = \frac{Z}{1 - A_v^{-1}}$

- **Proof**

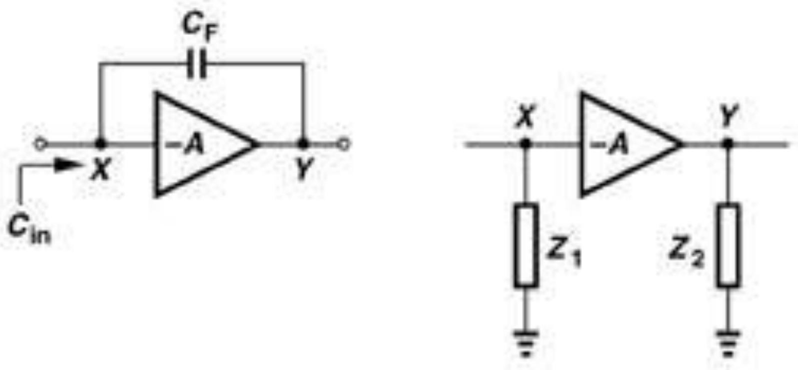
$$\frac{V_X - V_Y}{Z} = \frac{V_X}{Z_1} \Rightarrow Z_1 = \frac{Z}{1 - \frac{V_Y}{V_X}}$$

$$\frac{V_Y - V_X}{Z} = \frac{V_Y}{Z_2} \Rightarrow Z_2 = \frac{Z}{1 - \frac{V_X}{V_Y}}$$

Example 1

- **Calculate the input capacitance Cin:**

$$Z = \frac{1}{sC_F} \qquad Z_1 = \frac{\frac{1}{sC_F}}{1+A}$$



(a) (b)

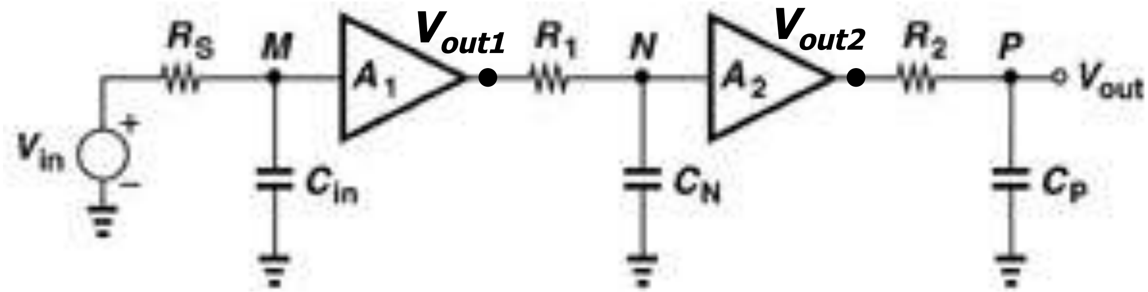
$$\Rightarrow \quad C_{in} = C_F(1+A)$$

$Av = \frac{V_Y}{V_X}$ **should be calculated at the frequency of interest.**

To simplify calculations we usually use low frequency value of Av.

Miller's theorem cannot be used simultaneously to calculate input-output transfer function and the output impedance.

Association of Poles with Nodes



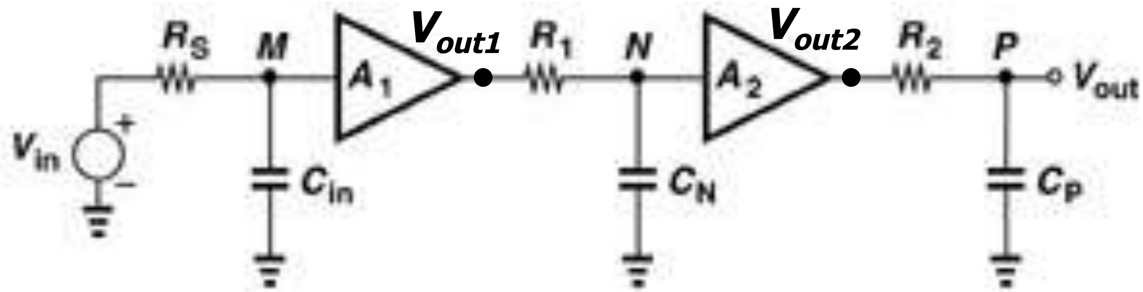
$$V_M(s) = \frac{V_{in}(s)}{R_S + \frac{1}{sC_{in}}} \frac{1}{sC_{in}} = \frac{V_{in}(s)}{1 + sR_S C_{in}}$$

$$V_N(s) = \frac{V_{out1}(s)}{1 + sR_1 C_N}$$

$$V_P(s) = \frac{V_{out2}(s)}{1 + sR_2 C_P}$$

$$\Rightarrow \quad \boxed{\frac{V_{out}}{V_{in}}(s) = \frac{A_1}{1 + sR_S C_{in}} \frac{A_2}{1 + sR_1 C_N} \frac{1}{1 + sR_2 C_P}}$$

Association of Poles with Nodes



$$\frac{V_{out}}{V_{in}}(s) = \frac{A_1}{1 + sR_S C_{in}} \frac{A_2}{1 + sR_1 C_N} \frac{1}{1 + sR_2 C_P}$$

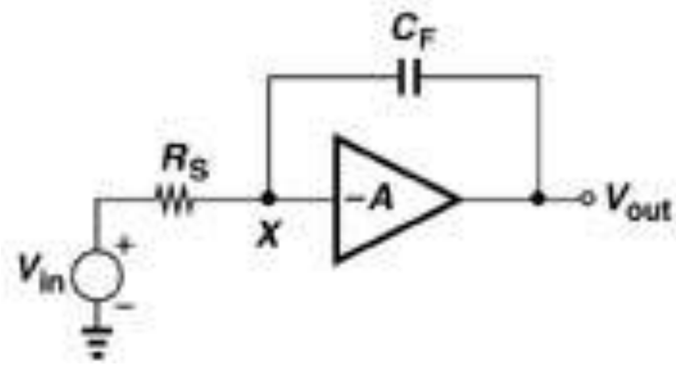
$$\omega_1 = \frac{1}{R_S C_{in}} \qquad \omega_2 = \frac{1}{R_1 C_N} \qquad \omega_3 = \frac{1}{R_2 C_P}$$

3 poles:

each determined by the total capacitance seen from each node to ground multiplied by the total resistance seen at the node to ground

Example 2

- **Calculate the pole associated with node X:**



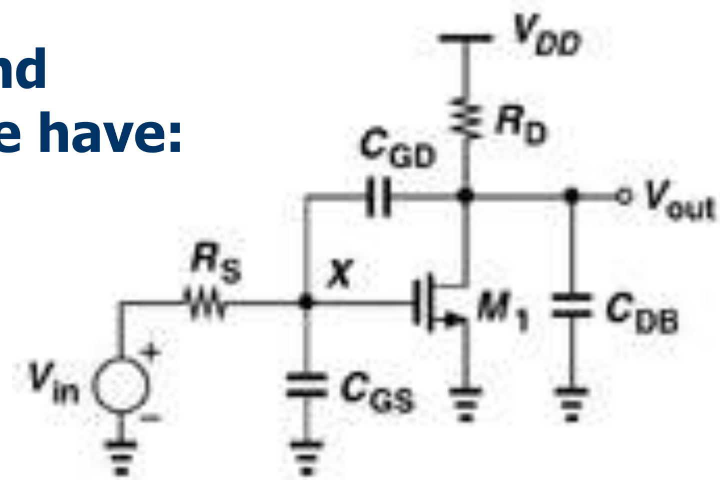
The total equivalent capacitance seen from X to ground: $C_X = C_F(1+A)$

The pole frequency:

$$\omega_X = \frac{1}{R_S C_X} = \frac{1}{R_S C_F (1+A)}$$

Common Source Stage

Neglecting channel length modulation and applying the Miller's theorem on C_{GD}, we have:



The total capacitance at node X:

$$C_X = C_{GS} + (1 - A_v) C_{GD}$$

where, $A_v = -g_m R_D$

The 1st pole frequency:

$$\omega_{p1} = \frac{1}{R_S \left(C_{GS} + (1 + g_m R_D) C_{GD} \right)}$$

The total capacitance at the output node:

$$C_{out} = C_{DB} + \left(1 - A_v^{-1}\right) C_{GD} \approx C_{DB} + C_{GD}$$

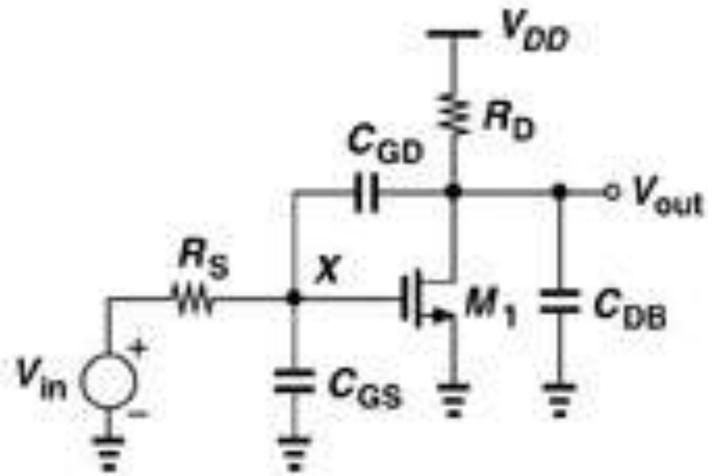
The 2nd pole frequency:

$$\omega_{p2} = \frac{1}{R_D \left(C_{DB} + C_{GD} \right)}$$

Common Source Stage

The transfer function:

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{-g_m R_D}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)}$$



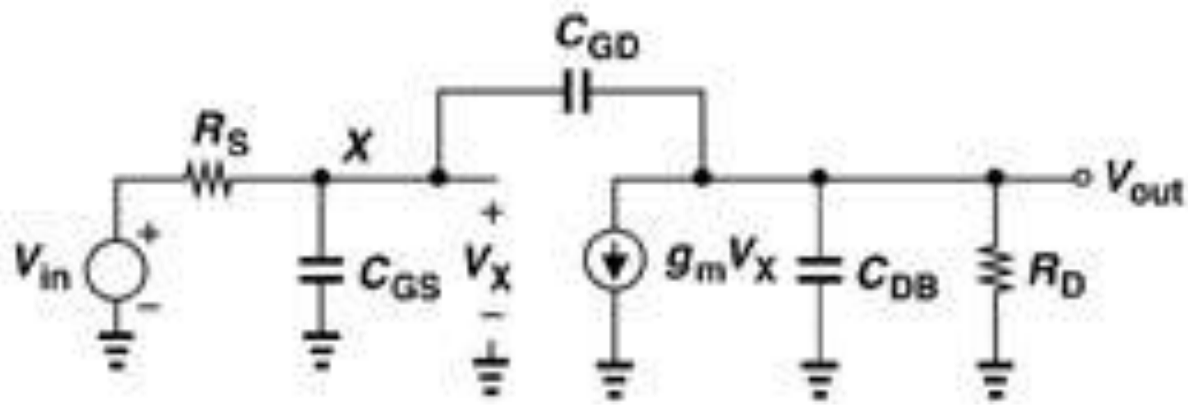
r_0 and any load capacitance can be easily included.

Sources of error (approximation):

- **we have not considered the existence of zeros in the circuit**
- **the amplifier gain varies with frequency**

Common Source : "exact " Transfer Function

To obtain the exact transfer function:



Applying Kirchoff Current Law (KCL):

$$\frac{V_X - V_{in}}{R_S} + sC_{GS}V_X + sC_{GD}\left(V_X - V_{out}\right) = 0$$

$$sC_{GD}\left(V_{out} - V_X\right) + g_m V_X + \left(sC_{DB} + \frac{1}{R_D}\right)V_{out} = 0$$

Common Source : "exact" 1st pole

After some manipulations, we get:

$$\frac{V_{out}}{V_{in}} = \frac{(sC_{GD} - g_m)R_D}{R_S R_D \xi s^2 + [R_S(1 + g_m R_D)C_{GD} + R_S C_{GD} + R_S C_{GS} + R_D(C_{GD} + C_{DB})]s + 1}$$

with $\xi = C_{GS}C_{GD} + C_{GS}C_{DB} + C_{GD}C_{DB}$

Writing the denominator as:

$$D = \left(1 + \frac{s}{\omega_{p1}}\right)\left(1 + \frac{s}{\omega_{p1}}\right) = \frac{s^2}{\omega_{p1}\omega_{p2}} + \left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right)s + 1$$

Assuming $|\omega_{p1}| << |\omega_{p2}|$

$$\Rightarrow \quad \boxed{\omega_{p1} \approx \frac{1}{R_S(1 + g_m R_D)C_{GD} + R_S C_{GD} + R_S C_{GS} + R_D(C_{GD} + C_{DB})}}$$

Compare this result with ω_{in} calculated using Miller's Theorem

Common Source : "exact" 2nd pole

$$\frac{V_{out}}{V_{in}} = \frac{(sC_{GD} - g_m)R_D}{R_S R_D \xi s^2 + \left[R_S(1+g_m R_D)C_{GD} + R_S C_{GD} + R_S C_{GS} + R_D(C_{GD}+C_{DB})\right]s + 1}$$

with $\xi = C_{GS}C_{GD} + C_{GS}C_{DB} + C_{GD}C_{DB}$

having $D = \frac{s^2}{\omega_{p1}\omega_{p2}} + \left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right)s + 1$

and $\omega_{p1} \approx \frac{1}{R_S(1+g_m R_D)C_{GD} + R_S C_{GD} + R_S C_{GS} + R_D(C_{GD}+C_{DB})}$

then $\omega_{p2} = \frac{1}{R_S R_D \xi}\frac{1}{\omega_{p1}}$

$$\Rightarrow \boxed{\omega_{p2} = \frac{R_S(1+g_m R_D)C_{GD} + R_S C_{GS} + R_D(C_{GD}+C_{DB})}{R_S R_D(C_{GS}C_{GD} + C_{GS}C_{DB} + C_{GD}C_{DB})}}$$

Comparison between "exact" and Miller's theorem

1st pole:

If

$$R_D\left(C_{GD}+C_{DB}\right)$$

is negligible

exact

$$\omega_{p1}=\frac{1}{R_S\left(C_{GS}+\left(1+g_mR_D\right)C_{GD}\right)+R_D\left(C_{GD}+C_{DB}\right)}$$

Miller

$$\omega_{p1}=\frac{1}{R_S\left(C_{GS}+\left(1+g_mR_D\right)C_{GD}\right)}$$

2nd pole:

exact

$$\omega_{p2}=\frac{R_S\left(1+g_mR_D\right)C_{GD}+R_SC_{GS}+R_D\left(C_{GD}+C_{DB}\right)}{R_SR_D\left(C_{GS}C_{GD}+C_{GS}C_{DB}+C_{GD}C_{DB}\right)}$$

$$if\ C_{GS}>>\left(1+g_mR_D\right)C_{GD}+\frac{R_D}{R_S}\left(C_{GD}+C_{DB}\right)$$

$$\omega_{p2}\approx\frac{C_{GS}}{R_D\left(C_{GS}C_{GD}+C_{GS}C_{DB}+C_{GD}C_{DB}\right)}$$

Miller

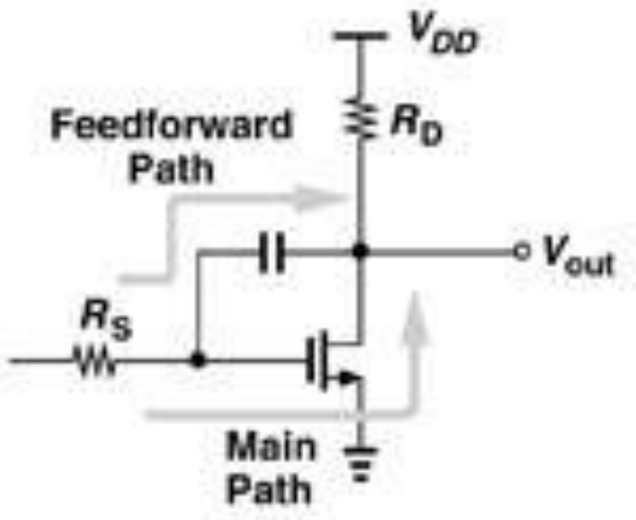
$$\omega_{p2}=\frac{1}{R_D\left(C_{DB}+C_{GD}\right)}$$

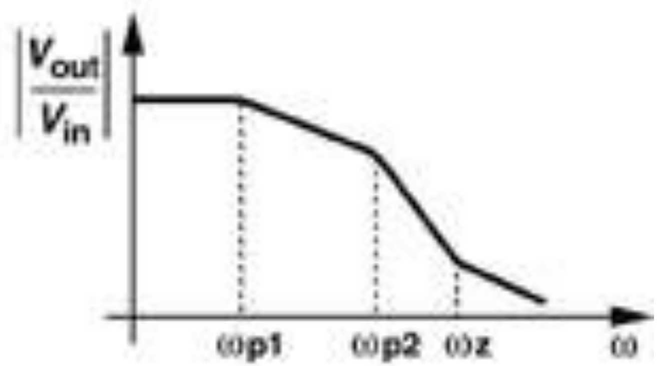
Common Source : transfer function zero

After some manipulations, we get:

$$\frac{V_{out}}{V_{in}} = \frac{\left(sC_{GD} - g_m\right)R_D}{R_S R_D \xi\, s^2 + \left[R_S\left(1+g_m R_D\right)C_{GD} + R_S C_{GD} + R_S C_{GS} + R_D\left(C_{GD}+C_{DB}\right)\right]s + 1}$$

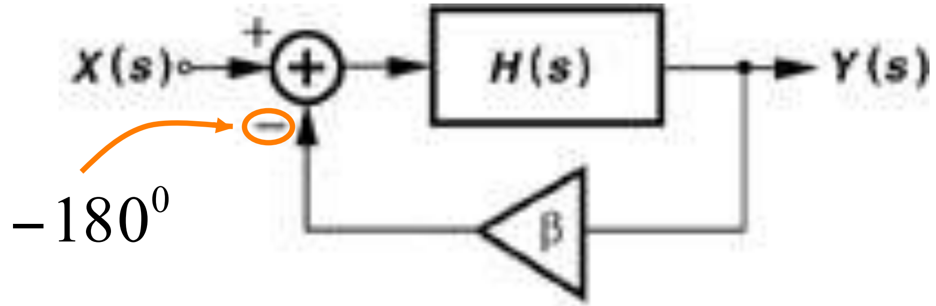
$$\omega_z = \frac{g_m}{C_{GD}}$$





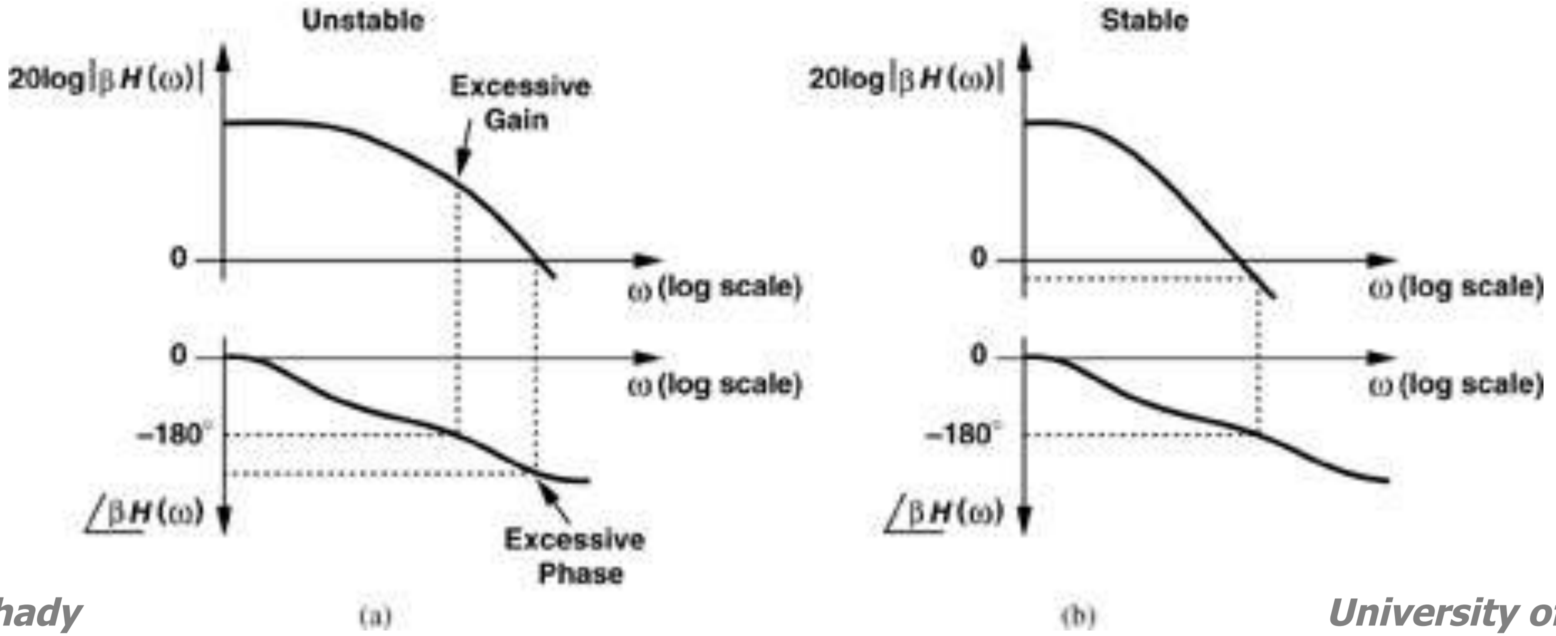
Stability and Frequency Compensation

$$\frac{Y(s)}{X(s)} = \frac{H(s)}{1+\beta H(s)}$$



Condition for oscillations

$$\beta H(j\omega) = -1 \quad \begin{cases} |\beta H(j\omega)| = 1 \\ \angle \beta H(j\omega) = -180^0 \end{cases}$$



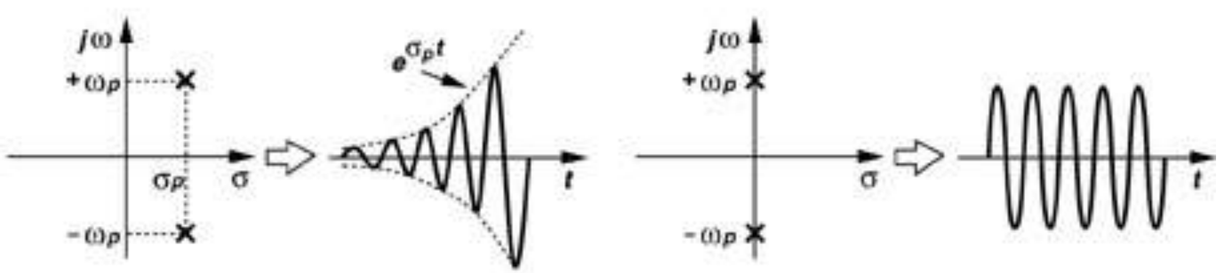
Bode Plot & Root Locus

Bode Plot:

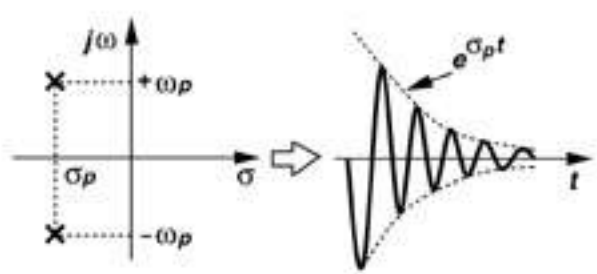
(1) The slope of the magnitude plot changes by
+ 20 dB/dec at every zero frequency
 - 20 dB/dec at every pole frequency

(2) For a pole (zero) frequency of ω_m , the phase begins to fall (rise) at $0.1\omega_m$, experiences a change - 45^0 (+ 45^0) at ω_m , and a change of -90^0 ($+90^0$) at $10\omega_m$.

Root Locus: $s_p = \sigma_p + j\omega_p$



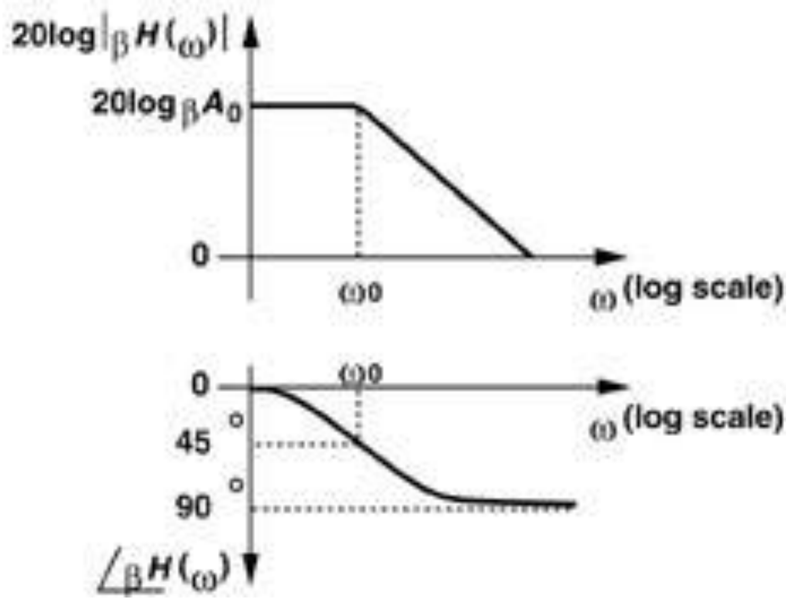
(a) (b)



One-Pole System

$$H(s) = \frac{A_0}{1 + s/\omega_0}$$

$$\frac{Y(s)}{X(s)} = \frac{H(s)}{1 + \beta H(s)}$$



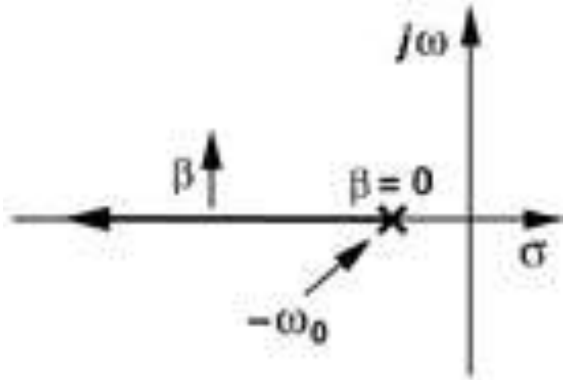
Bode Plot:

We plot $|\beta H(s)|$
and $\angle\beta H(s)$ **at** $s=j\omega$

A single pole cannot contribute to a phase shift greater than 90°
➡ **the system is unconditionally stable.**

Root Locus:

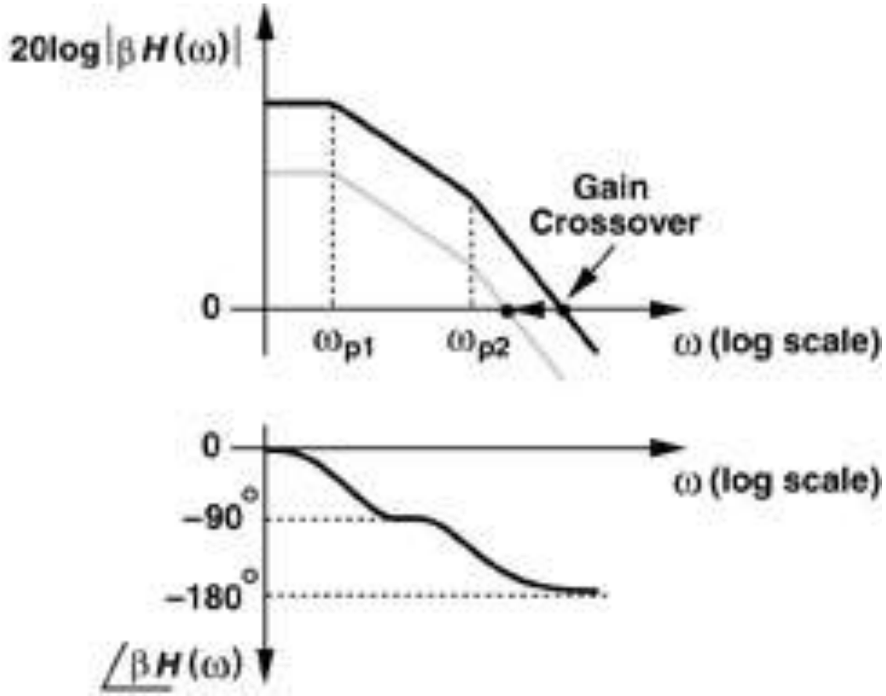
$$s_p = -\omega_0(1 + \beta A_0)$$



We plot the location of the poles as the loop gain varies

Two-Pole System

Bode Plot:



$\beta \downarrow$ ⇨ *Gain* $\downarrow$

⇨ *No Phase Change*

⇨ *More Stable System*

Two-Pole System

Root Locus:

$$H(s) = \frac{A_0}{(1+s/\omega_{p1})(1+s/\omega_{p2})}$$

$$\frac{Y(s)}{X(s)} = \frac{H(s)}{1+\beta\, H(s)}$$

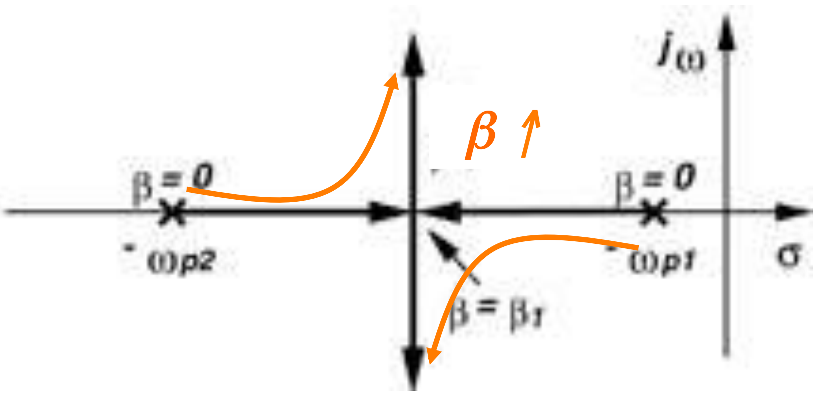
$$\frac{Y(s)}{X(s)} = \frac{A_0}{(1+s/\omega_{p1})(1+s/\omega_{p2})+\beta\, A_0}$$

$$= \frac{A_0\omega_{p1}\omega_{p2}}{s^2+(\omega_{p1}+\omega_{p2})s+(1+\beta\, A_0)\,\omega_{p1}\omega_{p2}}$$

$$s_{1,2} = -\frac{1}{2}(\omega_{p1}+\omega_{p2}) \pm \frac{1}{2}\sqrt{(\omega_{p1}+\omega_{p2})^2-4(1+\beta\, A_0)\omega_{p1}\omega_{p2}}$$

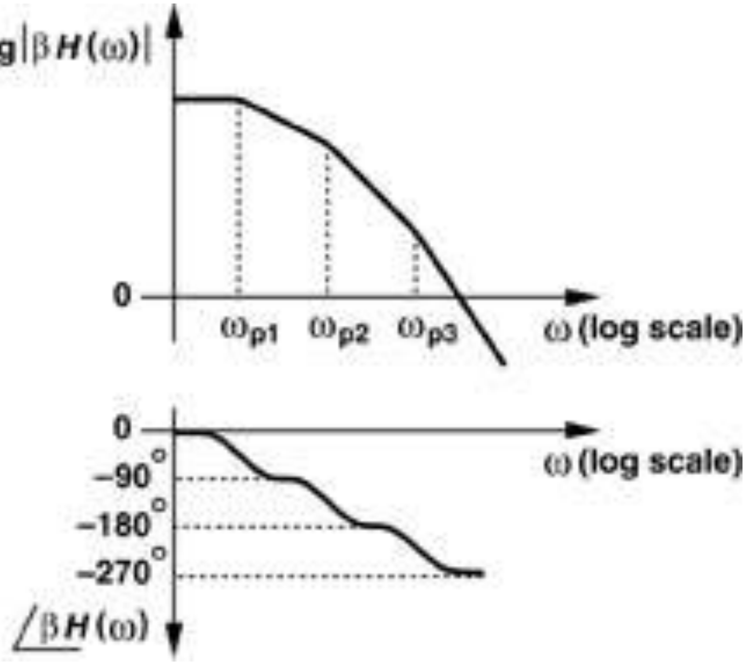
For $\beta = 0$ ➡ $s_{1,2} = -\omega_{p1}, -\omega_{p2}$

$$\beta_1 = -\frac{1}{A_0}\,\frac{(\omega_{p1}-\omega_{p2})^2}{4\,\omega_{p1}\omega_{p2}}$$



Three-Pole System

Additional poles and zeros impact the phase much more than the magnitude



Phase Margin

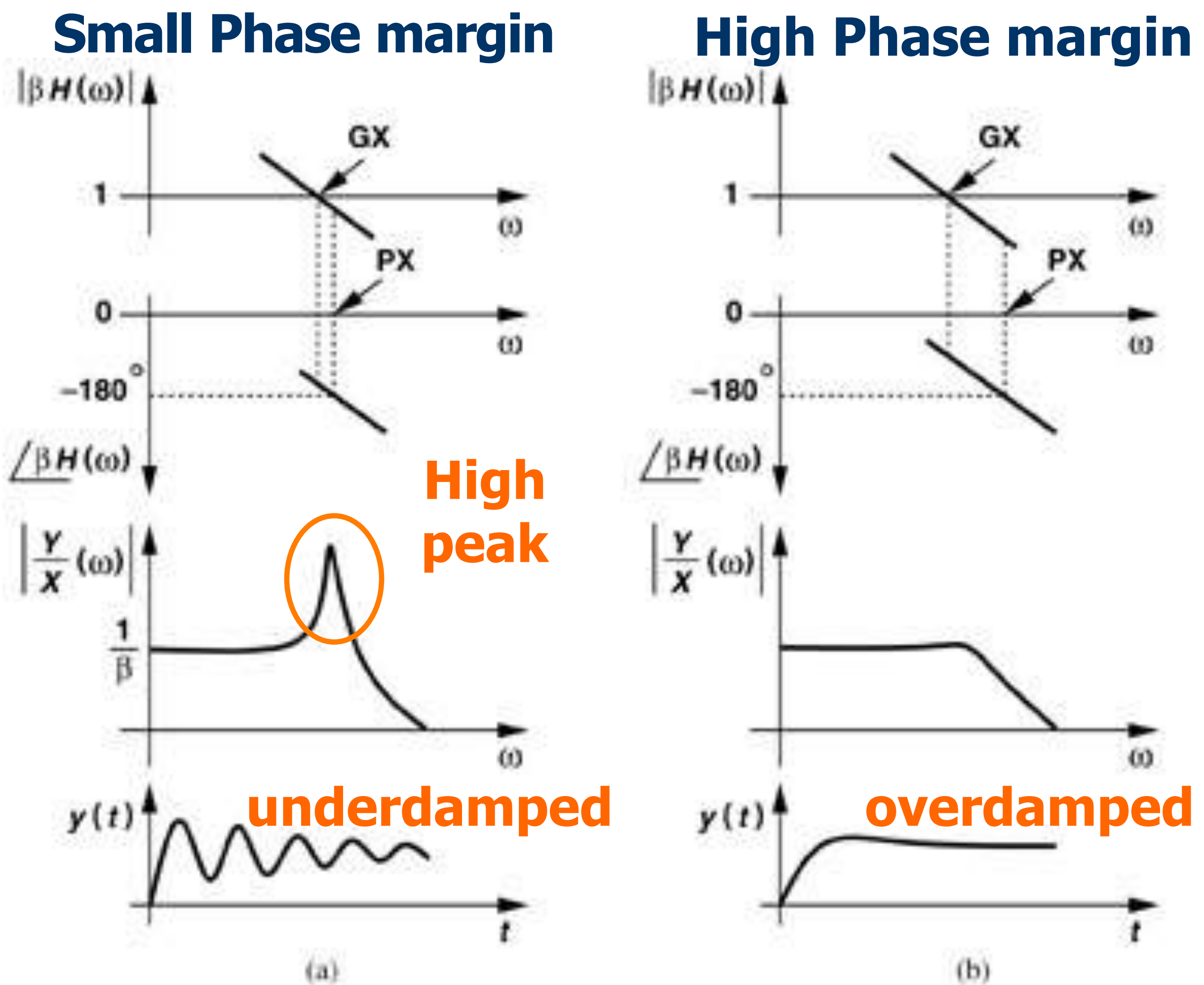
To ensure stability $|\beta H(s)|$ must drop to unity before $\angle\beta H(s)$ crosses -180°.

How far should PX be from GX ?

GX: Gain Crossover
PX: Phase Crossover

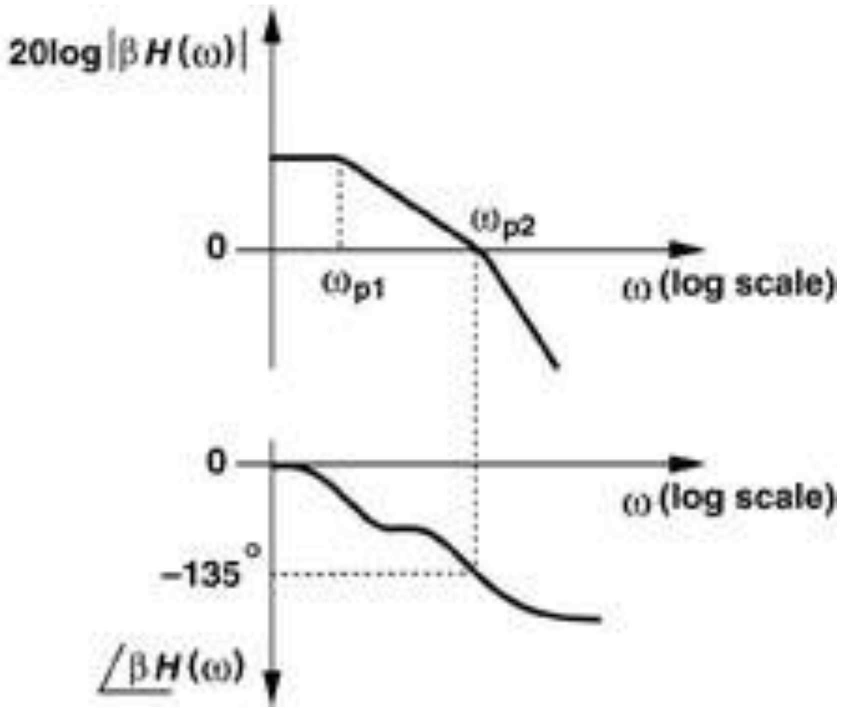
Frequency Response:

Step Response:



Example

A two-pole feedback system is designed such that $\left|\beta H(\omega_{p2})\right| = 1$ **and** $\left|\omega_{p1}\right| << \left|\omega_{p2}\right|$. **What is the phase margin ?**



Since $\angle\beta H(s)$ **reaches -135° at** $\omega = \omega_{p2}$

➡ **The phase margin is equal to 45°.**

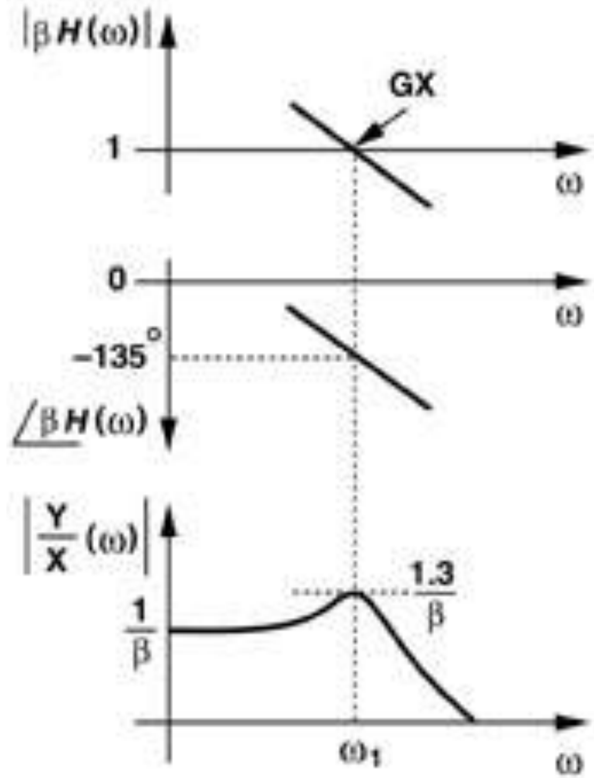
How much phase margin is adequate ?

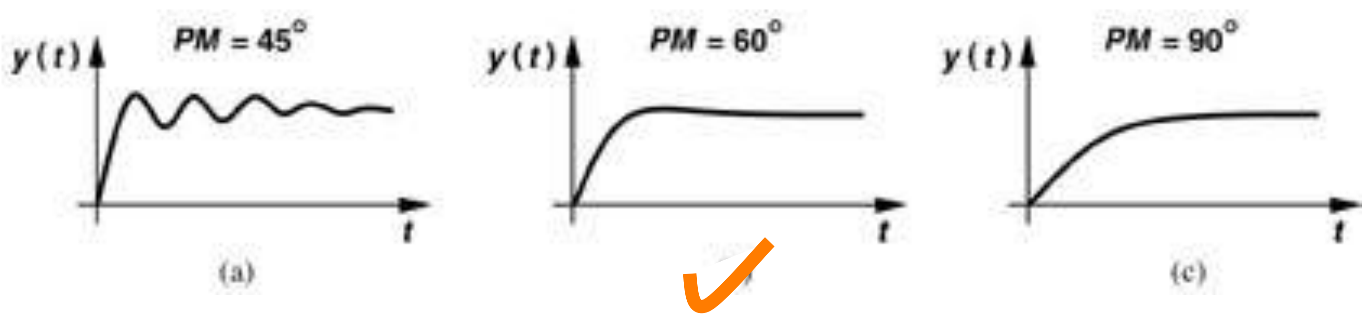
For PM=45° ⇨ $\beta H(\omega_1) = 1 \times \exp(-j135)$

$$\left.\frac{Y(s)}{X(s)}\right|_{s=j\omega_1} = \frac{1}{\beta}\,\frac{1\times\exp(-j135°)}{1+1\times\exp(-j135°)}$$

$$\left.\frac{Y(s)}{X(s)}\right|_{s=j\omega_1} = \frac{1}{\beta}\,\frac{-\sqrt{2}/2 - j\sqrt{2}/2}{1-\sqrt{2}/2 - j\sqrt{2}/2}$$

$$\left|\left.\frac{Y(s)}{X(s)}\right|_{s=j\omega_1}\right| = \frac{1.3}{\beta}$$

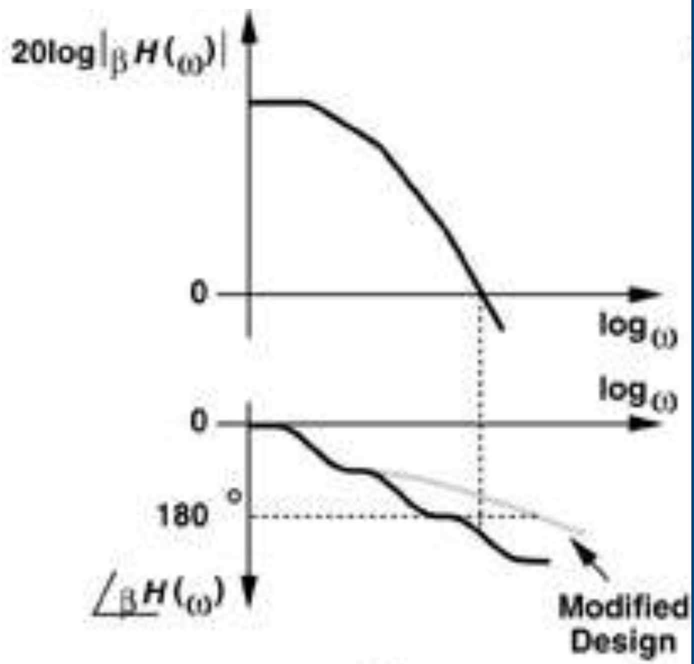




Frequency Compensation

Solution 1

➡ **Modify** $\angle \beta H(s)$

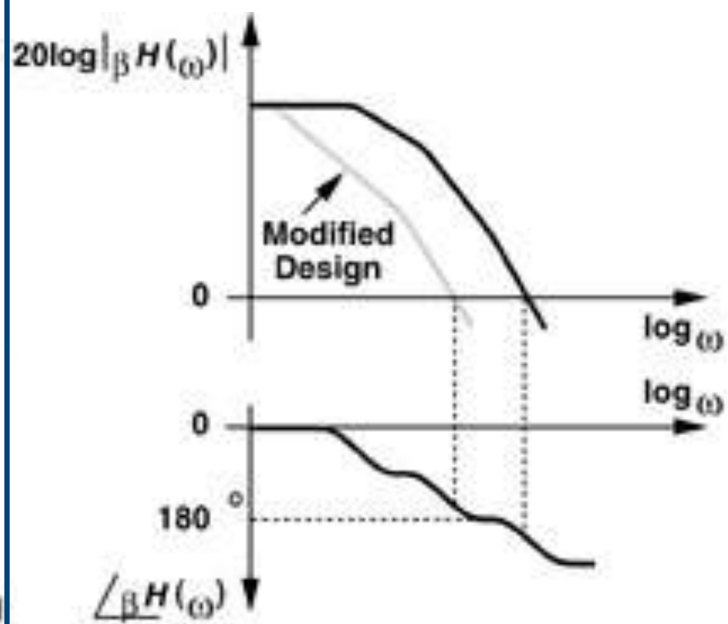


(a)

➡ **Poles ↓ ⇒ Stages ↓ ⇒ Gain ↓**

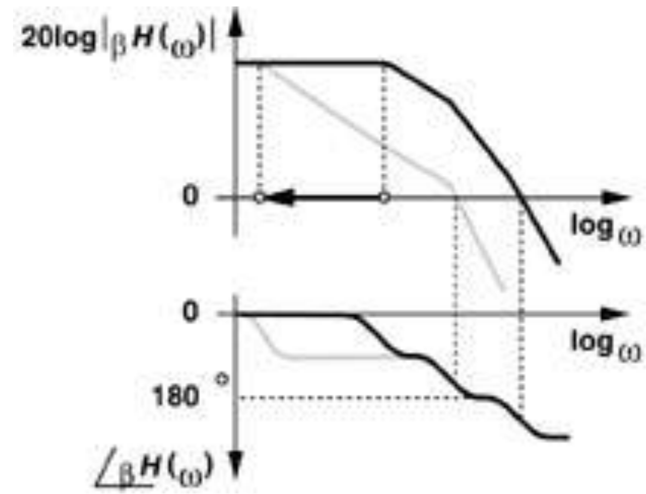
Solution 2

➡ **Modify** $|\beta H(s)|$



(b)

➡ **Reduces bandwidth**



Frequency Compensation:

- lower the frequency of the dominant pole

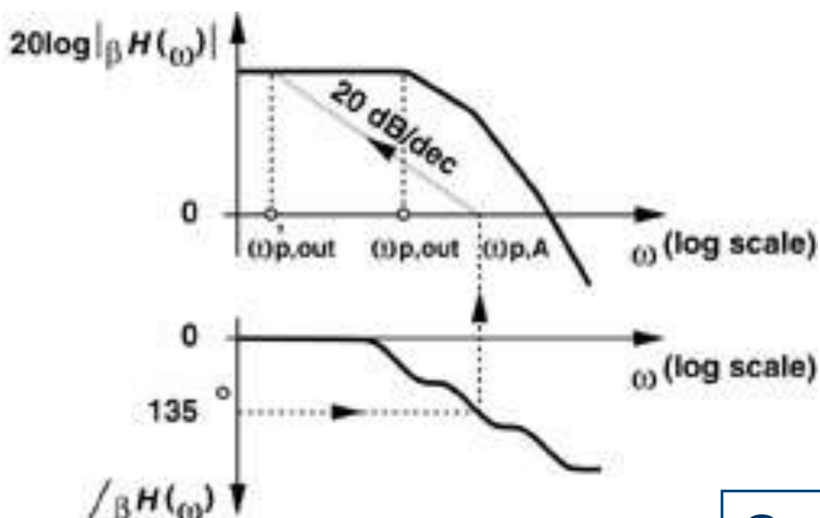
➡ **increase the load capacitance**

How much $\omega_{p,out}$ must be shifted down ?

Assume:

1- $\omega_{p,A} << \omega_{p,N}$ ➡ $\angle\beta\, H(\omega_{p,A}) = 135°$

2- required PM=45°



➡ **-The new dominant pole:** $\omega'_{p,out}$

-The load capacitance must be increased by a factor $\omega_{p,out} / \omega'_{p,out}$

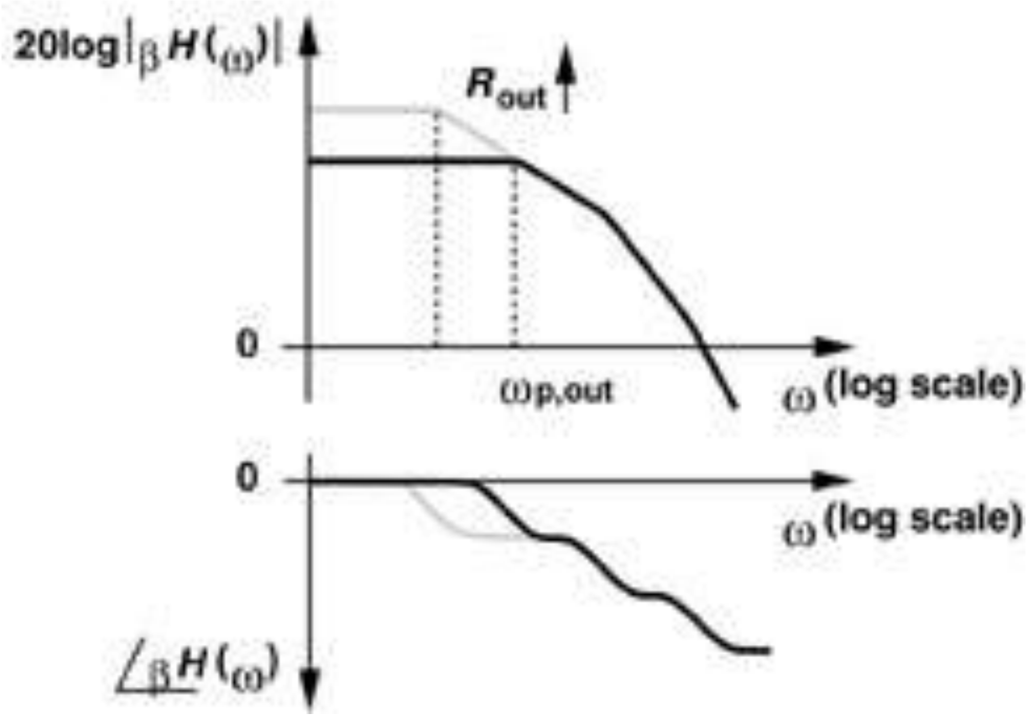
Op-Amp GBW = 1st non dominant pole

Is it possible to compensate using Rout ?

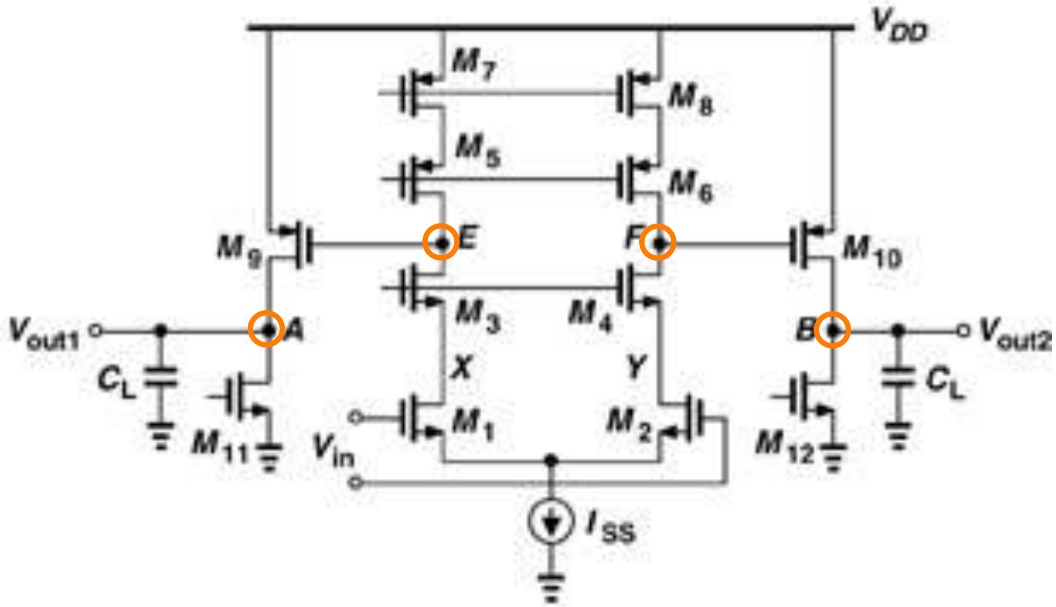
Although,

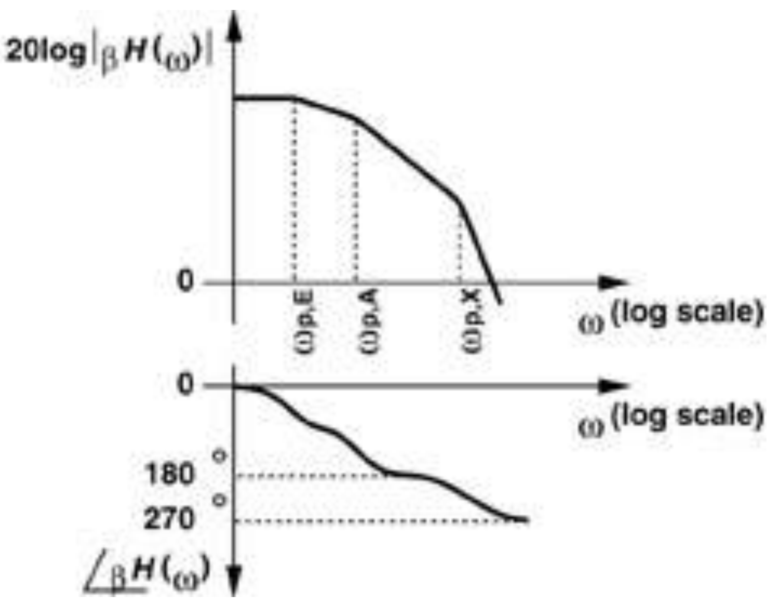
$$\omega_{p,out} = \frac{1}{R_{out} C_L}$$

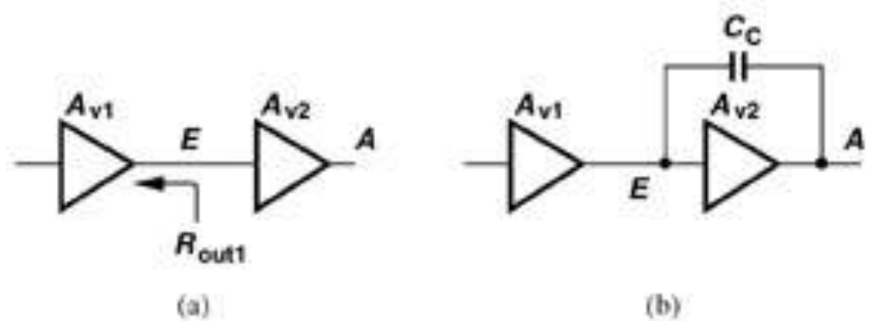
The answer is NO !



Compensation of 2 stage Op-Amps





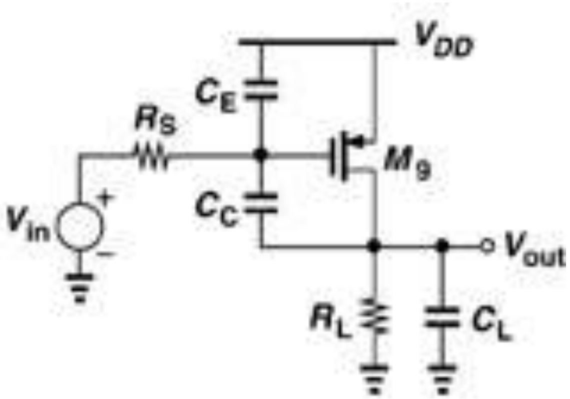


Total capacitance at node E:

$$C_E + (1 + A_{v2})C_C$$

2nd Stage

Common Source Amplifier:

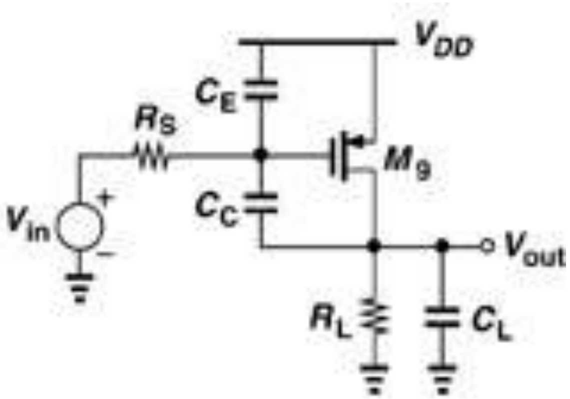


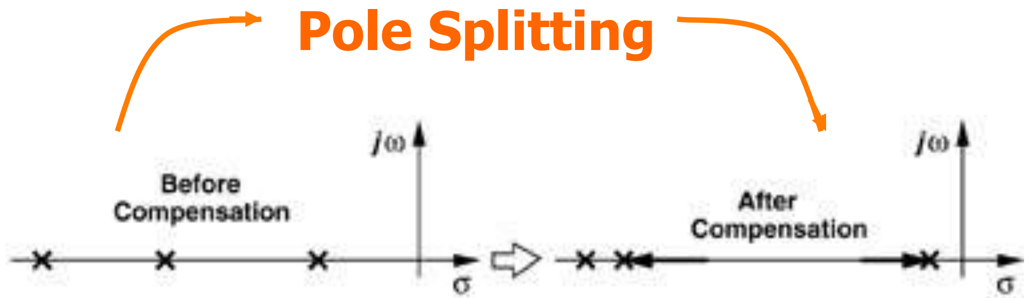
$$\omega_{p1} \approx \frac{1}{R_S\left[\left(1+g_{m9}R_L\right)\left(C_C+C_{GD9}\right)+C_E\right]+R_L\left(C_C+C_{GD9}+C_L\right)}$$

$$\omega_{p2} = \frac{R_S\left[\left(1+g_{m9}R_L\right)\left(C_C+C_{GD9}\right)+C_E\right]+R_SC_{GS}+R_L\left(C_C+C_{GD9}+C_L\right)}{R_SR_L\left[\left(C_C+C_{GD9}\right)C_E+\left(C_C+C_{GD9}\right)C_L+C_EC_L\right]}$$

Pole Splitting

Common Source Amplifier:





Before compensation:

$$\omega_{p1} \approx \frac{1}{R_S\left(C_E + \left(1 + g_{m9}R_L\right)C_{GD9}\right)}$$

$$\omega_{p2} \approx \frac{1}{R_L C_L}$$

After compensation:

$$\omega_{p1} \approx \frac{1}{R_S\left[C_E + \left(1 + g_{m9}R_L\right)\left(C_C + C_{GD9}\right)\right]}$$

$$\omega_{p2} \approx \frac{g_{m9}}{C_E + C_L}$$